CONTENTS

1. DIGITAL SIGNAL PROCESSING

- A signal is defined as any physical quantity that varies with time, space or another independent variable.
- A system is defined as a physical device that performs an operation on a signal.
- System is characterized by the type of operation that performs on the signal. Such operations are referred to as signal processing.

1.1 Advantages of DSP

1. A digital programmable system allows flexibility in reconfiguring the digital signal processing operations by changing the program. In analog redesign of hardware is required.
2. In digital accuracy depends on word length, floating Vs fixed point arithmetic etc. In analog depends on components.
3. Can be stored on disk.
4. It is very difficult to perform precise mathematical operations on signals in analog form but these operations can be routinely implemented on a digital computer using software.
5. Cheaper to implement.
6. Small size.
7. Several filters need several boards in analog, whereas in digital same DSP processor is used for many filters.

1.2 Disadvantages of DSP

1. When analog signal is changing very fast, it is difficult to convert digital form .(beyond 100KHz range)
2. w=1/2 Sampling rate.
3. Finite word length problems.
4. When the signal is weak, within a few tenths of millivolts, we cannot amplify the signal after it is digitized.

5. DSP hardware is more expensive than general purpose microprocessors & micro controllers.

6. Dedicated DSP can do better than general purpose DSP.

1.3 Applications of DSP

1. Filtering.
2. Speech synthesis in which white noise (all frequency components present to the same level) is filtered on a selective frequency basis in order to get an audio signal.
3. Speech compression and expansion for use in radio voice communication.
4. Speech recognition.
5. Signal analysis.
6. Image processing: filtering, edge effects, enhancement.
7. PCM used in telephone communication.
8. High speed MODEM data communication using pulse modulation systems such as FSK, QAM etc. MODEM transmits high speed (1200-19200 bits per second) over a band limited (3-4 KHz) analog telephone wire line.
9. Wave form generation.

1.4 Classification of Signals

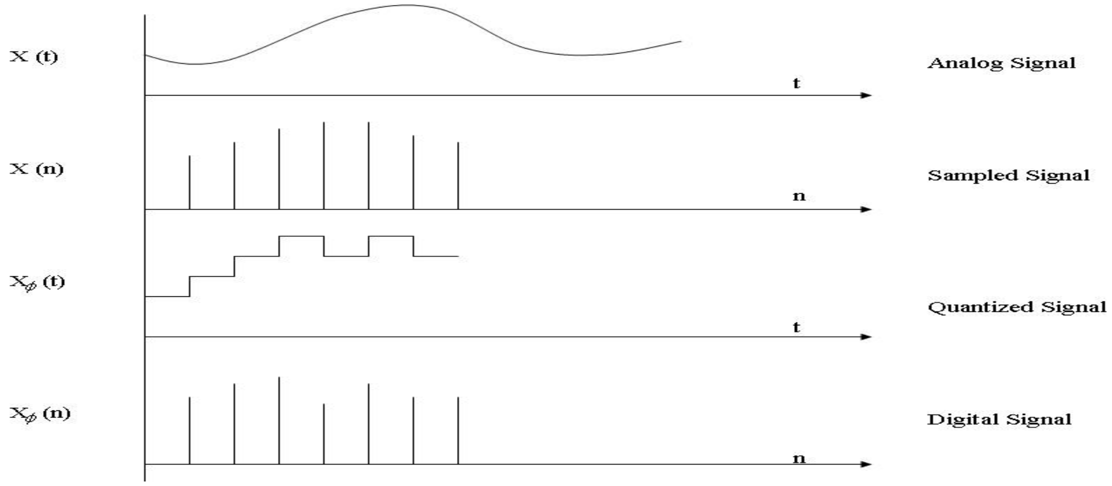
I. Based on Variables:

1. f(t)=5t : single variable
2. f(x,y)=2x+3y : two variables
3. S_1= A Sin(wt) : real valued signal
4. $S_2 = A\,e^{jwt}$: A Cos(wt)+j A Sin(wt) : Complex valued signal
5. $S_4(t)=\begin{bmatrix} S1(t) \\ S2(t) \\ S3(t) \end{bmatrix}$: Multichannel signal

Ex: due to earth quake, ground acceleration recorder

6. $I(x,y,t)=\begin{bmatrix} Ir(x,y,t) \\ Ig(x,y,t) \\ Ib(x,y,t) \end{bmatrix}$ multidimensional

II. Based on Representation:



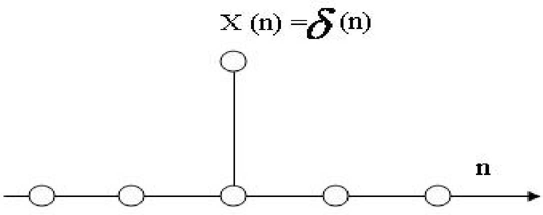
III. Based on duration.

1. right sided: x(n)=0 for n<N
2. left sided :x(n)=0 for n>N
3. causal : x(n)=0 for n<0
4. Anti causal : x(n)=0 for $n \geq 0$
5. Non causal : x(n)=0 for $|n|>N$

IV. Based on the Shape.

1. δ(n)=0 $\quad n \neq 0$

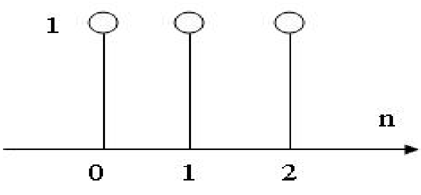
   =1 $\quad$ n=0



Unit Sample or Discrete time sample

2. u (n) =1 $\quad n \geq 0$

   =0 $\quad$ n<0



Arbitrary sequence can be represented as a sum of scaled, delayed impulses.

P (n) = $a_{-3}\delta$(n+3) +a_1 δ(u-1) +a_2 δ(u-2) +a_7 δ(u-7)

Or

x(n) = $\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

u(n) = $\sum_{k=-\infty}^{n} \delta(k)$ = δ(n) + δ(n-1)+ δ(n-2)…..

$= \sum_{k=0}^{\infty} \delta(n-k)$

3.Discrete pulse signals.

Rect (n/2N) =1 $\quad |n| \le$ N

= 0 $\quad$ else where.

5.Tri (n/N) = 1-$|n|$/N $\quad |n| \le$ N

= 0 $\quad$ else where.

1. Sinc (n/N)= Sa(n$\prod$/N) = Sin(n$\prod$/N) / (n$\prod$/N), Sinc(0)=1

Sinc (n/N) =0 at n=kN, k= ± 1, ± 2…

Sinc (n) = δ(n) for N=1; (Sin (n$\prod$) / n$\prod$=1= δ(n))

6.Exponential Sequence

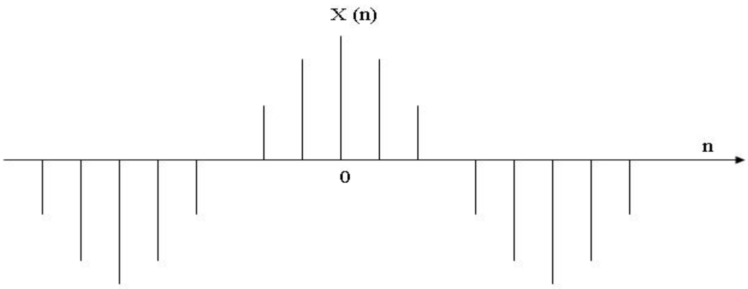
x (n) = A α^n

If A & α are real numbers, then the sequence is real. If 0<α<1 and A is +ve, then sequence values are +ve and decreases with increasing n.

For -1<α<0, the sequence values alternate in sign but again decreases in magnitude with increasing n. If $|\alpha|$>1, then the sequences grows in magnitude as n increases.

7.Sinusoidal Sequence

x(n) = A Cos(w_on+ϕ) for all n



8.Complex exponential sequence

If $\alpha = |\alpha| e^{jwo}$

$A = |A|\ e^{j\phi}$

$x(n) = |A|\ e^{j\phi}\ |\alpha|^n e^{jwon}$

$= |A|\,|\alpha|^n \mathrm{Cos}(w_o n+\phi) + j\ |A|\,|\alpha|^n \mathrm{Sin}(w_o n+\phi)$

If $\alpha > 1$, the sequence oscillates with exponentially growing envelope.

If $\alpha < 1$, the sequence oscillates with exponentially decreasing envelope.

So when discussing complex exponential signals of the form x(n)= A e^{jwon} or real sinusoidal signals of the form x(n)= A $\mathrm{Cos}(w_o n+\phi)$, we need only consider frequencies in a frequency internal of length $2\prod$ such as $\prod <$ Wo $< \prod$ or $0 \leq$ Wo<$2\prod$.

V. **Deterministic** (x (t) = α^t x (t) = A Sin(wt))

& Non-deterministic Signals. (Ex: Thermal noise.)

VI. **Periodic & non periodic based on repetition.**

VII. **Power & Energy Signals**

Energy signal: E = finite, P=0

- Signal with finite energy is called energy signal.
- Energy signal have zero signal power, since averaging finite energy over infinite time. All time limited signals of finite amplitude are energy signals.

Ex: one sided or two sided decaying. Damped exponentials, damped sinusoidal.

- x(t) is an energy signal if it is finite valued and x^2 (t) decays to zero fasten than $\frac{1}{|t|}$ as t $\rightarrow \infty$.

Power signal: $E = \infty$, $P \neq 0$, $P \neq \infty$ Ex: All periodic waveforms

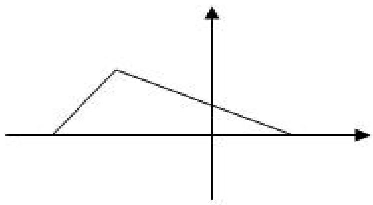
Neither energy nor power: $E=\infty$, $P=0$ Ex: $1/\sqrt{t}$ $t \geq 1$ $E=\infty$, $P=\infty$, Ex: t^n

VIII. Based on Symmetry

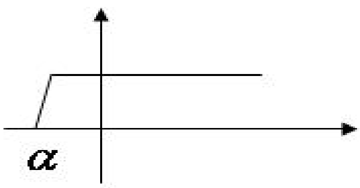
1.	Even	$x(n)=x_e(n)+x_o(n)$
2.	Odd	$x(-n)=x_e(-n)+x_o(-n)$
3.	Hidden	$x(-n)=x_e(n)-x_o(n)$
4.	Half-wave symmetry.	$x_e(n)=\frac{1}{2}[x(n)+x(-n)]$
		$x_o(n)=\frac{1}{2}[x(n)-x(-n)]$

Signal Classification by duration & Area.

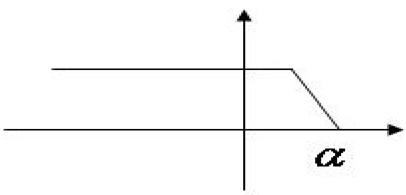
a. Finite duration: time limited.



b. Semi-infinite extent: right sided, if they are zero for $t < \alpha$ where α = finite



c. Left sided: zero for $t > \alpha$



Piecewise continuous: possess different expressions over different intervals.

Continuous: defined by single expressions for all time. $x(t) = \sin(t)$

Periodic: $x_p(t) = x_p(t \pm nT)$

For periodic signals $P = \frac{1}{T}\int_0^T |x(t)|^2\,dt$

$X_{rms} = \sqrt{P}$

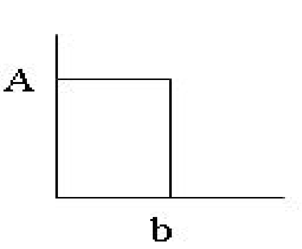
For non periodic

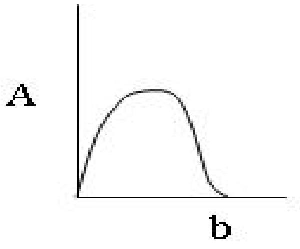
$$P = Lt \ \frac{1}{To}\int_0^T |x(t)|^2 \, dt$$

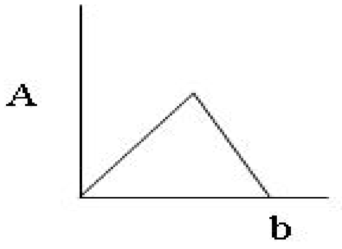
$$Xavg = Lt \int_0^{To} x(t)dt$$

$x(t) = A\cos(2\prod f_o t + \phi)$ $P=0.5\,A^2$

$x(t) = A\,e^{\pm j(2\prod f_o t + \phi)}$ $P=A^2$

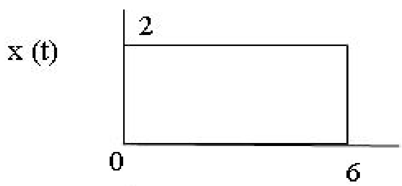






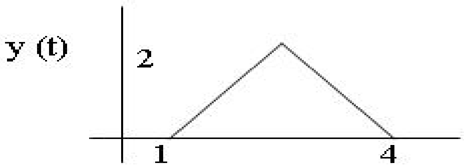
$E = A^2 b$ $E = \frac{1}{2}A^2 b$ $E = \frac{1}{3}A^2 b$

Q.

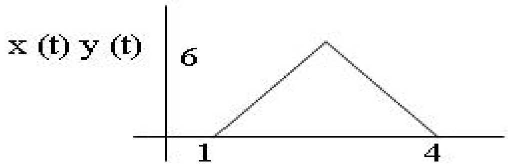


E=24

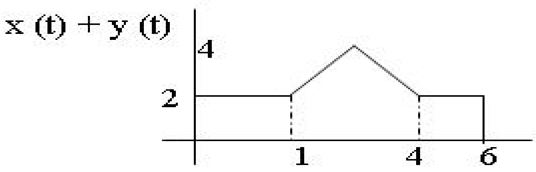
$$\int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha}$$



E=4 J

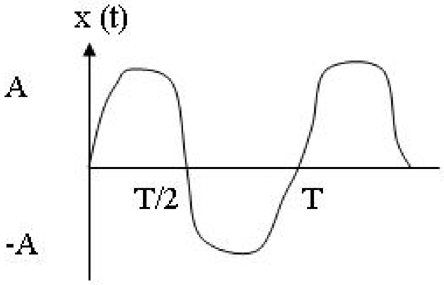


E=16 J



E=40 J

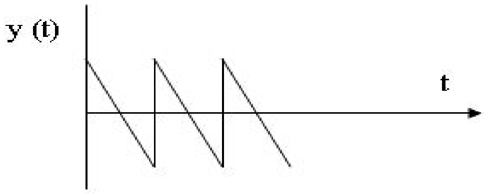
Q.



Ex = $\frac{1}{2}A^2\, 0.5T + \frac{1}{2}\,(-A)^2\, 0.5T = 0.5\, A^2\, T$

Px = $0.5\, A^2$

Q.



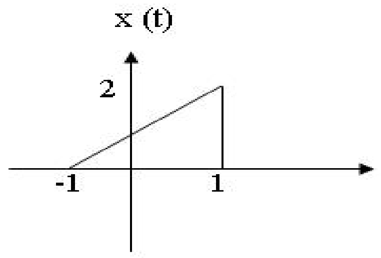
Ey = $[\frac{1}{3}A^2\, 0.5T]\, 2 = \frac{1}{3}\, A^2\, T$

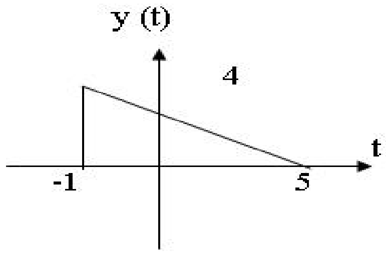
Py = $\frac{1}{3}\, A^2$

- x(t) = A e^{jwt} is periodic

Px = $\frac{1}{T}\int_0^T |x(t)|^2\, dt = A^2$

- x(2t -6): compressed by 2 and shifted right by 3 OR shifted by 6 and compressed by 2.
- x(1-t): fold x(t) & shift right by 1 OR shift right and fold.
- x(0.5t +0.5) Advance by 0.5 & stretched by 2 OR stretched by 2 & advance by 1.





y (t) = 2 x [- $\frac{(t-2)}{3}$] = 2 x[$\frac{-t}{3}+\frac{2}{3}$] 2 x(α t + β) ; 5$\alpha+\beta$=-1; -$\alpha+\beta$=1 => α = -1/3 ; β = 2/3

Area of symmetric signals over symmetric limits (-α , α)

Odd symmetry: $\int_{-\alpha}^{\alpha} x_0(t)\,dt = 0$

Even symmetry: $\int_{-\alpha}^{\alpha} x_e(t)\,dt = 2\int_{0}^{\alpha} x_e(t)\,dt$

Xe (t) +Ye (t): even symmetry.

Xe (t) Ye (t): even symmetry.

Xo (t) +Yo (t): odd symmetry.

Xo (t) Xo (t): even symmetry.

Xe (t) +Yo (t): no symmetry.

Xe (t) Yo (t): odd symmetry.

$X_e(n)= \frac{1}{2}[x(n)+x(-n)]$

$Xo(n) = \frac{1}{2}[x(n)-x(-n)]$

- Area of half-wave symmetry signal always zero.
- Half wave symmetry applicable only for periodic signal.
- F_0 = GCD (f_1,f_2)

T = LCM (T_1, T2)

- Y(t) = x_1(t) + x_2(t)

$P_y= P_{x1}+P_{x2}$

Y(t)rms = $\sqrt{Py}$

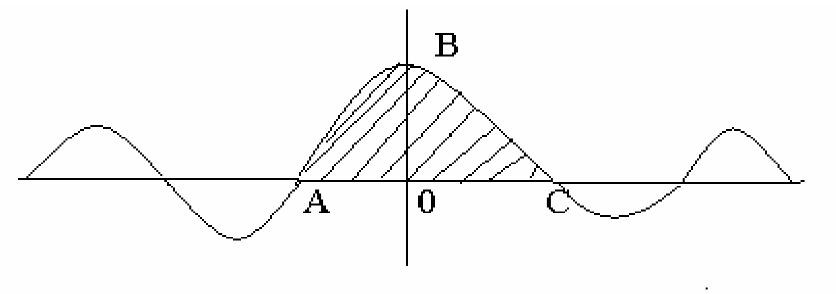
- U(0) = 0.5 is called as Heaviside unit step.
- X(t) = Sin(t) Sin(∏ t)

= 0.5 cos (1-∏)t – 0.5 cos (1+∏) t

W_1=1-∏

W_2=1+∏   almost periodic OR non periodic.

$P_x = 0.5[0.5^2 + 0.5^2] = 0.25$ W



Area of any sinc or Sinc ^{2} equals area of triangle ABC inscribed within the main lobe.

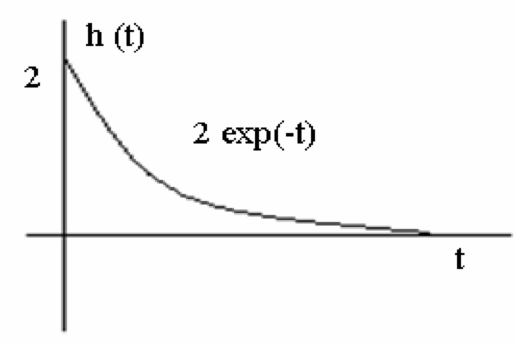
Even though the sinc function is square integrable (an energy signal) , it is not absolutely integrable(because it does not decay to zero faster than $\frac{1}{|t|}$)

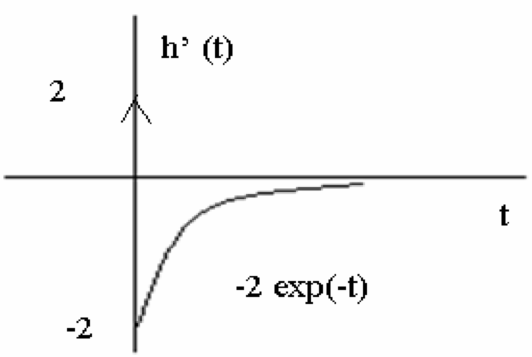
δ(t) = 0   t$\neq$0

$= \infty$ t=0 $\int_{-\infty}^{\infty} \delta(\tau) d\tau = 1$

An impulse is a tall narrow spike with finite area and infinite energy.

The area of impulse A δ(t) equals A and is called its strength. How ever its hight at t=0 is ∞.

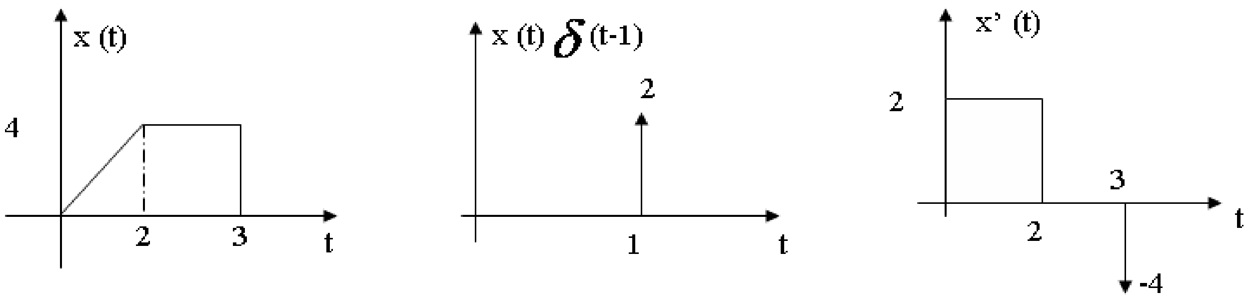




$= 2\ \delta(t) - 2e^{-t}\ u(t)$

$2\ e^{-t}\ \delta(t) = 2\ \delta(t)$

$\delta[\alpha[t-\beta]] = \left|\frac{1}{\alpha}\right|\delta(t-\beta)$

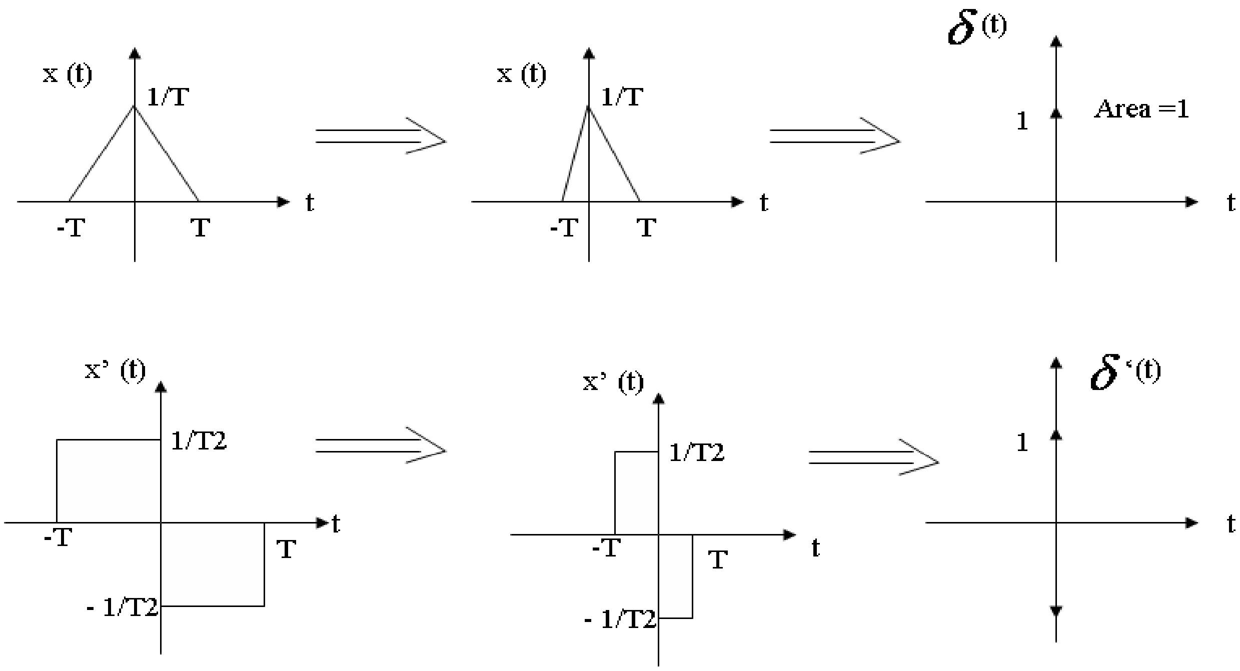


$I_2 = \int_{-4}^{2} \cos(2\prod t)\delta(2t+1)dt = \int_{-4}^{2} \cos(2\prod t)0.5\delta(t+0.5)dt = 0.5\ \cos(2\prod t)$ at $t=-0.5 = -0.5$

$x1(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kt_s) = \sum_{k=-\infty}^{\infty} x(kt_s)\ \delta(t-kt_s)$

x1(t) is not periodic.

The doublet



$\delta'(t) = 0 \qquad t \neq 0$

= undefined t=0 $\int_{-\infty}^{\infty} \delta'(t)dt = 0$ δ' (-t) = -δ' (t) then Odd function.

$\delta[\alpha[t-\beta]] = \left|\frac{1}{\alpha}\right|\delta(t-\beta)$

Differentiating on both sides

$\delta'[\alpha[t-\beta]] = \frac{1}{\alpha|\alpha|}\delta'(t-\beta)$

With α=-1

$\delta'(-t) = -\delta'(t)$

$\frac{d}{dt}[x(t)\delta(t-\alpha)]$= x' (t) δ(t-α) + x (t)δ' (t-α)

= x' (α) δ(t-α) + x (t)δ' (t-α)-----------1

Or

$\frac{d}{dt}[x(t)\delta(t-\alpha)] = \frac{d}{dt}[x(\alpha)\delta(t-\alpha)]$ = x (α)δ' (t-α) -----------2

1 = 2

x' (α) δ(t-α) + x (t)δ' (t-α) = x (α)δ' (t-α)

$\Rightarrow$ x (t)δ' (t-α) = x (α)δ' (t-α) - x' (α) δ(t-α)

$\int_{-\infty}^{\infty}$ x (t)δ' (t-α) dt = $\int_{-\infty}^{\infty}$ x (α)δ' (t-α) dt - $\int_{-\infty}^{\infty}$ x' (α) δ(t-α) dt

= 0- x' (α) = - x' (α)

Higher derivatives of δ(t) obey $\delta^n(t) = (-1)^n \delta^n(t)$ are alternately odd and even, and possess zero area. All are eliminating forms of the same sequence that generate impulses, provided their ordinary derivatives exits. None are absolutely integrable. The impulse is unique in being the only absolutely integrable function from among all its derivatives and integrals (step, ramp etc)

What does the signal x(t) = e^{-t} δ'(t) describe?

x(t) = δ' (t) – (-1) δ(t) = δ' (t) + δ(t)

$$I = \int_{-2}^{2}[(t-3)\delta(2t+2)] + 8\cos(\prod t)\delta'(t-0.5)]dt$$

$= 0.5\ (t-3)\big|_{t=-1}\ -\ 8\ \frac{d}{dt}[\cos \prod t]\big|_{t=0.5}$

= 23.1327 Answer.

1.5 Operation on Signals:

1. Shifting.

x(n) $\rightarrow$ shift right or delay = x(n-m)

x(n) $\rightarrow$ shift left or advance = x(n+m)

2. Time reversal or fold.

x(-n+2) is x(-n) delayed by two samples.

x(-n-2) is x(-n) advanced by two samples.

Or

x(n) is right shift x(n-2), then fold x(-n-2)

x(n) fold x(-n) shift left x(-(n+2)) = x(-n-2)

Ex:

x(n) = 2, 3 , $\underset{\uparrow}{4}$, 5, 6, 7 .

Find 1. y(n)=x(n-3) 2. x(n+2) 3. x(-n) 4. x(-n+1) 5. x(-n-2)

1. y(n)= x(n-3) = { $\underset{\uparrow}{0}$,2,3,4,5,6,7} shift x(n) right 3 units.

2. x(n+2) = { 2,3,4,5, $\underset{\uparrow}{6}$,7} shift x(n) left 2 units.

3. x(-n) = { 7,6,5, $\underset{\uparrow}{4}$,3,2} fold x(n) about n=0.

4. x(-n+1) = { 7,6, $\underset{\uparrow}{5}$,4,3,2} fold x(n), delay by 1.

5. x(-n-2) = { 7,6,5,4,3,2} fold x(n), advanced by 2.

3. a. Decimation.

Suppose x(n) corresponds to an analog signal x(t) sampled at intervals Ts. The signal y(n) = x(2n) then corresponds to the compressed signal x(2t) sampled at Ts and contains only alternate samples of x(n)(corresponding to x(0), x(2), x(4)…). We can also obtain directly from x(t) (not in compressed version). If we sample it at intervals 2Ts (or at a sampling rate Fs = $\frac{1}{2Ts}$). This means a two fold reduction in the sampling rate. Decimation by a factor N is equivalent to sampling x(t) at intervals NTs and implies an N-fold reduction in the sampling rate.

b. Interpolation.

y(n) = x(n/2) corresponds to x(t) sampled at Ts/2 and has twice the length of x(n) with one new sample between adjacent samples of x(n).

The new sample value as ‘0’ for Zero interpolation.

The new sample constant = previous value for step interpolation.

The new sample average of adjacent samples for linear interpolation.

Interpolation by a factor of N is equivalent to sampling x(t) at intervals Ts/N and implies an N-fold increase in both the sampling rate and the signal length.

Ex:

$$\{\underset{\uparrow}{1}, 2, 6, 4, 8\} \xrightarrow[n \to 2n]{\text{Decimation}} \{\underset{\uparrow}{1}, 6, 8\} \xrightarrow[n \to n/2]{\text{Step interpolation}} \{\underset{\uparrow}{1}, 1, 6, 6, 8, 8\}$$

$$\{\underset{\uparrow}{1}, 2, 6, 4, 8\} \xrightarrow[n \to n/2]{\text{Step interpolation}} \{\underset{\uparrow}{1}, 1,2,2,6, 6,4,4,8, 8\} \xrightarrow[n \to 2n]{\text{Decimation}} \{\underset{\uparrow}{1}, 2, 6, 4, 8\}$$

Since Decimation is indeed the inverse of interpolation, but the converse is not necessarily true. First Interpolation & Decimation.

Ex: $x(n) = \{ \underset{\uparrow}{1}1, 2, 5, -1\}$

$x(n/3) = \{ 1,0,0,\underset{\uparrow}{2}2,0,0,5,0,0,-1,0,0\}$ Zero interpolation.

$= \{ 1,1,1, \underset{\uparrow}{2},2,2,5,5,5,-1,-1,-1\}$ Step interpolation.

$= \{ 1,\frac{4}{3}, \frac{5}{3}, \underset{\uparrow}{2}, 3,4,5,3,1,-1, -\frac{2}{3},-\frac{1}{3}\}$ Linear interpolation.

4. Fractional Delays.

It requires interpolation (N), shift (M) and Decimation (n): $x\left(n - \frac{M}{N}\right) = x\left(\frac{(Nn-M)}{N}\right)$

$x(n) = \{2, 4, \underset{\uparrow}{6}, 8\}$, find $y(n)=x(n-0.5) = x\left(\frac{2n-1}{2}\right)$

$g(n) = x(n/2) = \{2, 2, 4, 4, \underset{\uparrow}{6}, 6, 8, 8\}$ for step interpolation.

$h(n) = g(n-1) = x(\frac{n-1}{2}) = \{2, 2, 4, \underset{\uparrow}{4}, 6, 6, 8, 8\}$

$y(n) = h(2n) = x(n-0.5) = x(\frac{2n-1}{2}) = \{2, \underset{\uparrow}{4}, 6, 8\}$

OR

$g(n) = x(n/2) = \{2,3,4,5, \underset{\uparrow}{6}, 7,8,4\}$ linear interpolation.

$h(n) = g(n-1) = \{2,3,4, \underset{\uparrow}{5}, 6, 7,8,4\}$

$g(n) = h(2n) = \{3,5,7,4\}$

1.6 Classification of Systems

1. a. Static systems or memory less system. (Non Linear / Stable)

Ex. $y(n) = a\, x(n)$

$= n\, x(n) + b\, x^3(n)$

$= [x(n)]^2 \quad = a(n-1)\, x(n)$

$y(n) = \tau\, [x(n), n]$

If its o/p at every value of 'n' depends only on the input x(n) at the same value of 'n'

Do not include delay elements. Similarly to combinational circuits.

b. Dynamic systems or memory.

If its o/p at every value of 'n' depends on the o/p till (n-1) and i/p at the same value of 'n' or previous value of 'n'.

Ex. $y(n) = x(n) + 3\, x(n-1)$

$= 2\, x(n) - 10\, x(n-2) + 15\, y(n-1)$

Similar to sequential circuit.

2. Ideal delay system. (Stable, linear, memory less if nd=0)

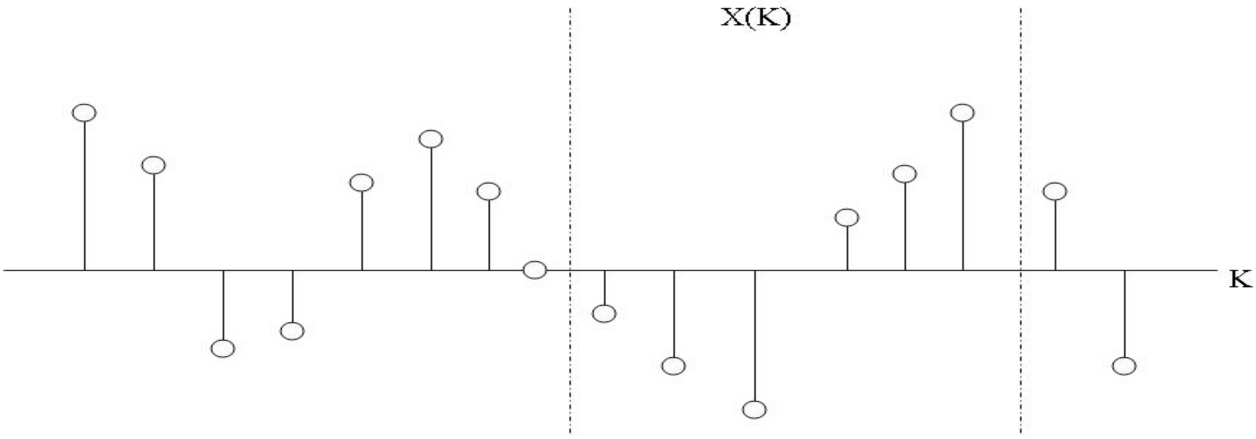
Ex. $y(n) = x(n-nd)$

nd is fixed = +ve integer.

3. **Moving average system. (LTIV ,Stable)**

$$y(n) = 1/ (m_1+m_2+1) \sum_{k=-m1}^{m2} x(n-k)$$

This system computes the n^{th} sample of the o/p sequence as the average of (m_1+m_2+1) samples of input sequence around the n^{th} sample.



If M1=0; M2=5

$$y(7) = 1/6 \left[\sum_{k=0}^{5} x(7-k) \right]$$

$$= 1/6 [x(7) + x(6) + x(5) + x(4) + x(3) + x(2)]$$

$$y(8) = 1/6 [x(8) + x(7) + x(6) + x(5) + x(4) + x(3)]$$

So to compute y (8), both dotted lines would move one sample to right.

4. **Accumulator. (Linear , Unstable)**

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$= \sum_{k=-\infty}^{n-1} x(k) + x(n)$$

$$= y(n-1) + x(n)$$

x(n) = { …0,3,2,1,0,1,2,3,0,….}

y(n) = { …0,3,5,6,6,7,9,12,12…}

O/p at the n^{th} sample depends on the i/p's till n^{th} sample

Ex:

x(n) = n u(n) ; given y(-1)=0. i.e. initially relaxed.

$$y(n) = \sum_{k=-\infty}^{-1} x(k) + \sum_{k=0}^{n} x(k)$$

$$= y(-1) + \sum_{k=0}^{n} x(k) = 0 + \sum_{k=0}^{n} n = \frac{n(n+1)}{2}$$

5. Linear Systems.

If $y_1(n)$ & $y_2(n)$ are the responses of a system when $x_1(n)$ & $x_2(n)$ are the respective inputs, then the system is linear if and only if

$$\tau[x1(n) + x2(n)] = \tau[x1(n)] + \tau[x2(n)]$$

$$= y_1(n) + y_2(n) \quad \text{(Additive property)}$$

$$\tau[ax(n)] = a\,\tau[x(n)] = a\, y(n) \quad \text{(Scaling or Homogeneity)}$$

The two properties can be combined into principle of superposition stated as

$$\tau[ax1(n) + bx2(n)] = a\,\tau[x1(n)] + b\,\tau[x2(n)]$$

Otherwise non linear system.

6. Time invariant system.

Is one for which a time shift or delay of input sequence causes a corresponding shift in the o/p sequence.

$y(n-k) = \tau[x(n-k)]$ TIV

$\neq$ TV

7. Causality.

A system is causal if for every choice of n_o the o/p sequence value at index $n = n_o$ depends only on the input sequence values for $n \leq n_o$.

y(n) = x(n) + x(n-1) causal.

y(n) = x(n) + x(n+2) + x(n-4) non causal.

8. Stability.

For every bounded input $|x(n)| \leq B_x < \infty$ for all n, there exists a fixed +ve finite value By such that $|y(n)| \leq B_y < \infty$.

1.7 PROPERTIES OF LTI SYSTEM.

1. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$

y(n) = $\tau\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$ for linear

$\sum_{k=-\infty}^{\infty} x(k)\ \tau[\delta(n-k)]$ for time invariant

$\sum_{k=-\infty}^{\infty} x(k)h(n-k) = x(n) * h(n)$

Therefore o/p of any LTI system is convolution of i/p and impulse response.

$$y(n_o) = \sum_{k=-\infty}^{\infty} h(k)x(no-k)$$

$$= \sum_{k=-\infty}^{-1} h(k)x(no-k) + \sum_{k=0}^{\infty} h(k)x(no-k)$$

$$= h(-1)\,x(n_0+1) + h(-2)\,x(n_0+2)\ldots\ldots\ldots+h(0)\,x(n_0) + h(1)\,x(n_0-1) + \ldots.$$

y(n) is causal sequence if h(n) =0 $n<0$

y(n) is anti causal sequence if h(n) =0 $n \geq 0$

y(n) is non causal sequence if h(n) =0 $|n|>N$

Therefore causal system y(n) = $\sum_{k=0}^{\infty} h(k)x(n-k)$

If i/p is also causal y(n) = $\sum_{k=0}^{n} h(k)x(n-k)$

2. Convolution operation is commutative.

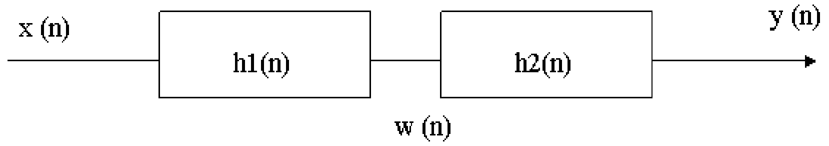
$x(n) * h(n) = h(n) * x(n)$

3. Convolution operation is distributive over additive.

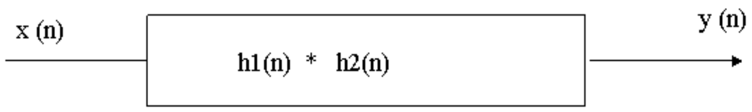
$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

4. Convolution property is associative.

$x(n) * h_1(n) * h_2(n) = [x(n) * h_1(n)] * h_2(n)$



5 y(n) = h_2 * w(n) = h2(n)*h1(n)*x(n) = h3(n)*x(n)



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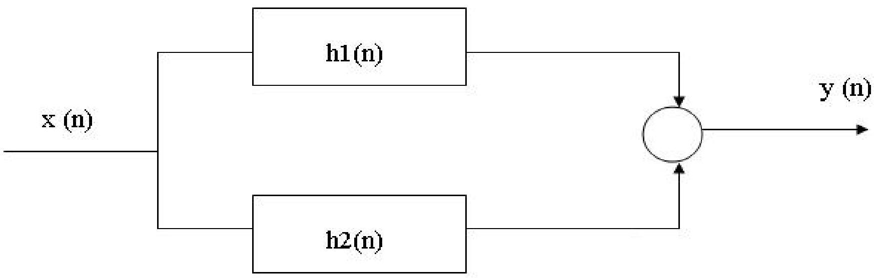


$h(n) = h_1(n) + h_2(n)$

7 LTI systems are stable if and only if impulse response is absolutely summable.

$$|y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)x(n-k)\right| \le \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)|$$

Since x (n) is bounded $|x(n)| \le b_x < \infty$

$$\therefore\ |y(n)| \le B_x \sum_{k=-\infty}^{\infty} |h(k)|$$

$\therefore S = \sum_{k=-\infty}^{\infty} |h(k)|$ is necessary & sufficient condition for stability.

8 $\delta(n) * x(n) = x(n)$

9 Convolution yields the zero state response of an LTI system.

10 The response of LTI system to periodic signals is also periodic with identical period.

$y(n) = h(n) * x(n)$

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{\infty} h(k)x(n-k+N)$$

put n-k = m

$$= \sum_{m=-\infty}^{\infty} h(n-m)x(m+N)$$

$$= \sum_{m=-\infty}^{\infty} h(n-m)x(m)$$

m=k

$$= \sum_{k=-\infty}^{\infty} h(n-k)x(k) \quad = \text{ y(n) (Ans)}$$

Q. y (n)-0.4 y(n-1) =x (n). Find causal impulse response? h(n)=0 n<0.

h(n) = 0.4 h(n-1) + $\delta(n)$

h(0) = 0.4 h(-1) + $\delta(0)$ =1

h(1) = 0.4 h(0) = 0.4

h(2) = 0.4^2

h(n) = 0.4^n for $n \geq 0$

Q. y(n)-0.4 y(n-1) = x(n). find the anti-causal impulse response? h(n)=0 for $n \geq 0$

h(n-1) = 2.5 [h(n)- $\delta(n)$]

h(-1) = 2.5 [h(0)- $\delta(0)$] = -2.5

h(-2) = -2.5^2 . …….. h(n) = -2.5^n valid for $n \leq -1$

Q. x(n)={1,2,3} y(n)={3,4} Obtain difference equation from i/p & o/p information

y(n) + 2 y(n-1) + 3 y(n-2) = 3 x(n) + 4 x(n-1) (Ans)

Q. x(n) = {4,4,}, y(n)= x(n)- 0.5x(n-1). Find the difference equation of the inverse system. Sketch the realization of each system and find the output of each system.

Solution:

The original system is y(n)=x(n)-0.5 x(n-1)

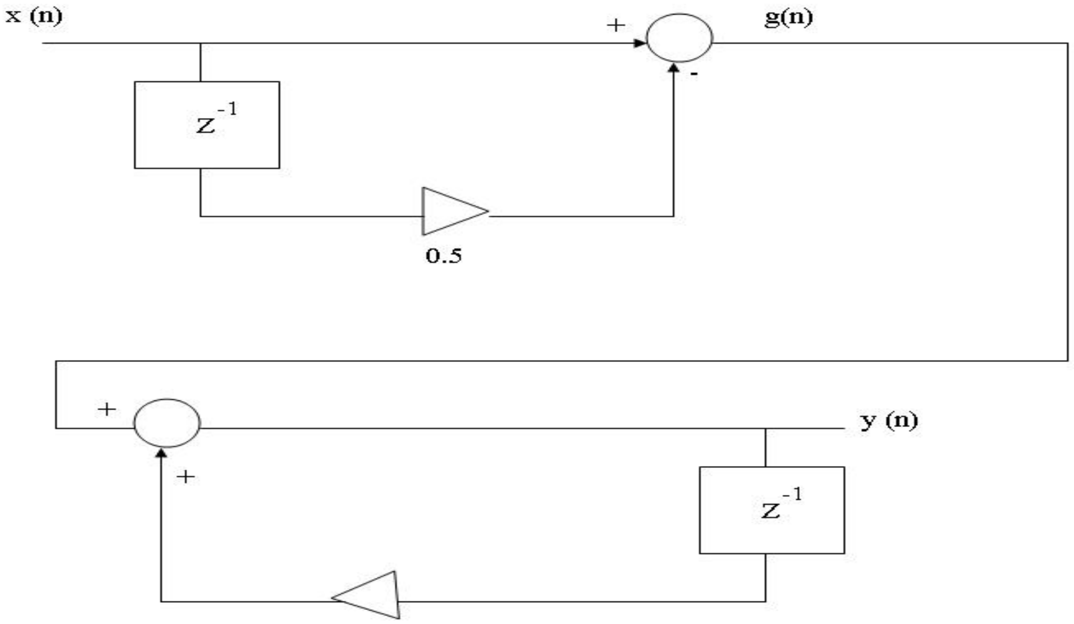
The inverse system is x(n)= y(n)-0.5 y(n-1)

y (n) = x (n) – 0.5 x(n-1)

$Y(z) = X(z)\,[1-0.5Z^{-1}]$

$\frac{Y(z)}{X(z)} = 1-0.5\,Z^{-1}$

System



Inverse System

$y(n) - 0.5\,y(n-1) = x(n)$

$Y(z)\,[1-0.5\,Z^{-1}] = X(z)$

$\frac{Y(z)}{X(z)} = [1-0.5\,Z^{-1}]^{-1}$

$g(n) = 4\,\delta(n) - 2\delta(n-1) + 4\delta(n-1) - 2\delta(n-2) = 4\delta(n) + 2\delta(n-1) - 2\delta(n-2)$

$y(n) = 0.5\,y(n-1) + 4\delta(n) + 2\delta(n-1) - 2\delta(n-2)$

$y(0) = 0.5y(-1) + 4\delta(0) = 4$

$y(1) = 4$

$y(2) = 0.5\,y(1) - 2\delta(0) = 0$

$y(n) = \{4, 4\}$ same as i/p.

Non Recursive filters	Recursive filters
$y(n) = \sum_{k=-\infty}^{\infty} a_k\, x(n-k)$ for causal system $= \sum_{k=0}^{\infty} a_k\, x(n-k)$	$y(n) = \sum_{k=0}^{N} a_k\, x(n-k) - \sum_{k=1}^{N} b_k\, y(n-k)$ Present response is a function of

For causal i/p sequence $y(n) = \sum_{k=0}^{N} a_k x(n-k)$ Present response depends only on present i/p & previous i/ps but not future i/ps. It gives FIR o/p.	the present and past N values of the excitation as well as the past N values of response. It gives IIR o/p but not always. $y(n) - y(n-1) = x(n) - x(n-3)$

Q. $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$ Find the given system is stable or not?

Let $x(n) = \delta(n)$

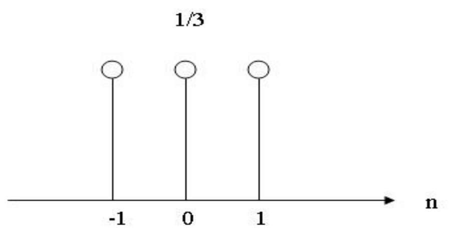
$h(n) = \frac{1}{3}[\delta(n+1) + \delta(n) + \delta(n-1)]$

$h(0) = \frac{1}{3}$

$h(-1) = \frac{1}{3}$

$h(1) = \frac{1}{3}$

$S = \sum h(n) < \infty$ therefore Stable.



Q. $y(n) = a\,y(n-1) + x(n)$ given $y(-1) = 0$

Let $x(n) = \delta(n)$

$h(n) = y(n) = a\,y(n-1) + \delta(n)$

$h(0) = a\,y(-1) + \delta(0) = 1 = y(0)$

$h(1) = a\,y(0) + \delta(1) = a$

$h(2) = a\,y(1) + \delta(2) = a^2$ . . . . $h(n) = a^n u(n)$ stable if a<1.

$y(n-1) = \frac{1}{a}[y(n) - x(n)]$

$y(n) = \frac{1}{a}[\, y(n+1) - x(n+1)]$

$y(-1) = \frac{1}{a}[\, y(0) - x(0)]=0$

$y(-2) = 0$

Q. $y(n) = \frac{1}{n+1}\, y(n-1) + x(n)$ for $n \geq 0$

$= 0$ otherwise. Find whether given system is time variant or not?

Let $x(n) = \delta(n)$

$h(0) = 1\, y(-1) + \delta(0) = 1$

$h(1) = ½\, y(0) + \delta(1) = ½$

$h(2) = 1/6$

$h(3) = 1/24$

if $x(n) = \delta(n-1)$

$y(n) = h(n-1)$

$h(n-1) = y(n) = \frac{1}{n+1}\, h(n-2) + \delta(n-1)$

n=0 $h(-1) = y(0) = 1 \times 0+0 = 0$

n=1 $h(0) = y(1) = ½ \times 0 + \delta(0)= 1$

n=2 $h(1) = y(2) = 1/3 \times 1 + 0 = 1/3$

$h(2) = 1/12$

$\therefore h(n, 0) \neq h(n,1)$ $\therefore$ TV

Q. $y(n) = 2n\, x(n)$ Time varying

Q. $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1)]$ Linear

Q. $y(n) = 12\, x(n-1) + 11\, x(n-2)$ TIV

Q. $y(n) = 7\, x^2(n-1)$ non linear

Q. $y(n) = x^2(n)$ non linear

Q. $y(n) = n^2\, x(n+2)$ linear

Q. $y(n) = x(n^2)$ linear

Q. $y(n) = e^{x(n)}$ non linear

Q. y (n) = $2^{x(n)}$ x (n) non linear, TIV

(If the roots of characteristics equation are a magnitude less than unity. It is a necessary & sufficient condition)

Non recursive system, or FIR filter are always stable.

Q. y (n) + 2 y^2(n) = 2 x(n) – x(n-1) non linear, TIV

Q. y (n) - 2 y (n-1) = $2^{x(n)}$ x (n) non linear, TIV

Q. y (n) + 4 y (n) y (2n) = x (n) non linear, TIV

Q. y (n+1) – y (n) = x (n+1) is causal

Q. y (n) - 2 y (n-2) = x (n) causal

Q. y (n) - 2 y (n-2) = x (n+1) non causal

Q. y (n+1) – y (n) = x (n+2) non causal

Q. y (n-2) = 3 x (n-2) is static or Instantaneous.

Q. y (n) = 3 x (n-2) dynamic

Q. y (n+4) + y (n+3) = x (n+2) causal & dynamic

Q. y (n) = 2 x (αn)

If α =1 causal, static

α <1 causal, dynamic

α >1 non causal, dynamic

$\alpha \neq 1$ TV

Q. y (n) = 2(n+1) x (n) is causal & static but TV.

Q. y (n) = x (-n) TV

1.8 Solution of linear constant-co-efficient difference equation

Q. y(n)-3 y (n-1) – 4 y(n-2) = 0 determine zero-input response of the system;

Given y(-2) =0 & y(-1) =5

Let solution to the homogeneous equation be

$y_h(n) = \lambda^n$

$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$

$\lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$

$\lambda = -1, 4$

$y_h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n \quad = \quad C_1(-1)^n + C_2 4^n$

$y(0) = 3y(-1) + 4y(-2) = 15$

$\therefore C_1 + C_2 = 15$

$y(1) = 3y(0) + 4y(-1) = 65$

$\therefore -C_1 + 4C_2 = 65$ Solve: $C_1 = -1$ & $C_2 = 16$

$y(n) = (-1)^{n+1} + 4^{n+2}$ (Ans)

If it contain multiple roots $y_h(n) = C_1 \lambda_1^n + C_2 n \lambda_1^n + C_3 n^2 \lambda_1^n$

or $\lambda_1^n [C_1 + nC_2 + n^2 C_3 \ldots]$

Q. Determine the particular solution of $y(n) + a_1 y(n-1) = x(n)$

$x(n) = u(n)$

Let $y_p(n) = k\, u(n)$

$k\, u(n) + a_1 k\, u(n-1) = u(n)$

To determine the value of k, we must evaluate this equation for any $n \geq 1$

$k + a_1 k = 1$

$$k = \frac{1}{1 + a1}$$

$$y_p(n) = \frac{1}{1 + a1} u(n) \quad \text{Ans}$$

$x(n)$	$y_p(n)$
1. A	K
2. Am^n	Km^n
3. An^m	$K_o n^m + K_1 n^{m-1} + \ldots K_m$
4. $A \operatorname{Cos} w_o n$ or $A \operatorname{Sin} w_o n$	$K_1 \operatorname{Cos} w_o n + K_2 \operatorname{Sin} w_o n$

Q. $y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n) \qquad x(n) = 2^n \quad n \geq 0$

Let $y_p(n) = K2^n$

$$K2^n u(n) = \frac{5}{6} K\, 2^{n-1} u(n-1) - \frac{1}{6} K\, 2^{n-2} u(n-2) + 2^n u(n)$$

For $n \geq 2$

$4K = \frac{5}{6}(2K) - \frac{1}{6} K + 4$ Solve for $K = 8/5$

$\therefore \quad y_p(n) = \frac{8}{5}2^n$ Ans

Q. y(n) – 3 y(n-1) - 4 y(n-2) = x(n) + 2x(n-1) Find the h(n) for recursive system.

We know that $y_h(n) = C_1(-1)^n + C_2\,4^n$

$y_p(n) = 0$ when $x(n) = \delta(n)$

for n=0

$y(0) - 3y(-1) - 4y(-2) = \delta(0) + 2\delta(-1)$

$\therefore y(0) = 1$

$y(1) = 3\,y(0) + 2 = 5$

$C_1 + C_2 = 1$

$-C_1 + C_2 = 5$ Solving $C_1 = -\frac{1}{5}$; $C_2 = \frac{6}{5}$

$\therefore h(n) = [-\frac{1}{5}(-1)^n + \frac{6}{5}4^n]\,u(n)$ Ans

OR

$h(n) - 3h(n-1) - 4h(n-2) = \delta(n) + 2\delta(n-1)$

$h(0) = 1$

$h(1) = 3h(0) + 2 = 5$

plot for h(n) in both the methods are same.

Q. $y(n) - 0.5\,y(n-1) = 5\cos 0.5n\Pi$ $n \geq 0$ with $y(-1) = 4$

$y_h(n) = \lambda^n$

$\lambda^n - 0.5\lambda^{n-1} = 0$

$\lambda^{n-1}[\lambda - 0.5] = 0$

$\lambda = 0.5$

$\therefore y_h(n) = C(0.5)^n$

$y_p(n) = K_1\cos 0.5n\Pi + K_2\sin 0.5n\Pi$

$y_p(n-1) = K_1\cos 0.5(n-1)\Pi + K_2\sin 0.5(n-1)\Pi$

$= -K_1\sin 0.5n\Pi - K_2\cos 0.5n\Pi$

$y_p(n) - 0.5\,y_p(n-1) = 5\cos 0.5\,n\Pi$

$= (K_1 + 0.5\ K_2) \cos 0.5\ n\Pi - (0.5\ K_1 - K_2) \sin 0.5n\Pi$

$K_1 + 0.5\ K_2 = 5$

$0.5\ K_1 - K_2 = 0$ Solving we get: $K_1 = 4$ & $K_2 = 2$

$\therefore\ y_p(n) = 4 \cos 0.5\ n\Pi + 2 \sin 0.5n\Pi$

The final response

$y(n) = C\ (0.5)^n + 4 \cos 0.5\ n\Pi + 2 \sin 0.5n\Pi$

with $y(-1) = 4$

$4 = 2C - 2$

i.e. $C = 3$

$\therefore\ y(n) = 3\ (0.5)^n + 4 \cos 0.5\ n\Pi + 2 \sin 0.5n\Pi$ for $n \geq 0$

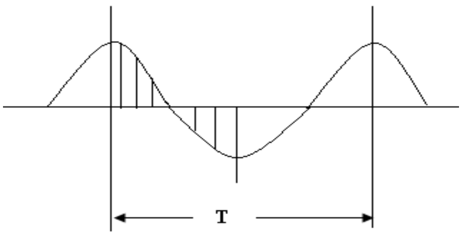
1.9 Concept of frequency in continuous-time and discrete-time.

1) $x_a(t) = A \cos(\Omega t)$

$x(nTs) = A \cos(\Omega nTs)$

$= A \cos(wn)$

$w = \Omega Ts$



Ω = rad / sec   w = rad / Sample

F = cycles / sec   f = cycles / Sample

2) A Discrete- time – sinusoid is periodic only of its f is a Rational number.

$x(n+N) = x(n)$

$\cos 2\pi f_0 (n+N) = \cos 2\pi f_0 n$

$2\pi f_0 N = 2\pi K \Rightarrow f_0 = \frac{K}{N}$

Ex: $A \cos\left(\frac{\Pi}{6}\right) n$

$w = \frac{\Pi}{6} = 2\pi f$

$f = \frac{1}{12}$ N=12 Samples/Cycle ; Fs= Sampling Frequency; Ts = Sampling Period

Q. Cos (0.5n) is not periodic

Q. x (n) = 5 Sin (2n)

$2\pi f = 2 \Rightarrow f = \frac{1}{\Pi}$ Non-periodic

Q. x (n) = 5 Cos (6πn)

$2\pi f = 6\pi \Rightarrow f = 3$ N=1 for K=3 Periodic

Q. x (n) = 5 Cos $\frac{6\Pi n}{35}$

$2\pi f = \frac{6\Pi}{35} \Rightarrow f = \frac{3}{35}$ for N=35 & K=3 Periodic

Q. x (n) = Sin (0.01πn)

$2\pi f = 0.01\pi \Rightarrow f = \frac{0.01}{2}$ for N=200 & K=1 Periodic

Q. x (n) = Cos (3πn) for N=2 Periodic

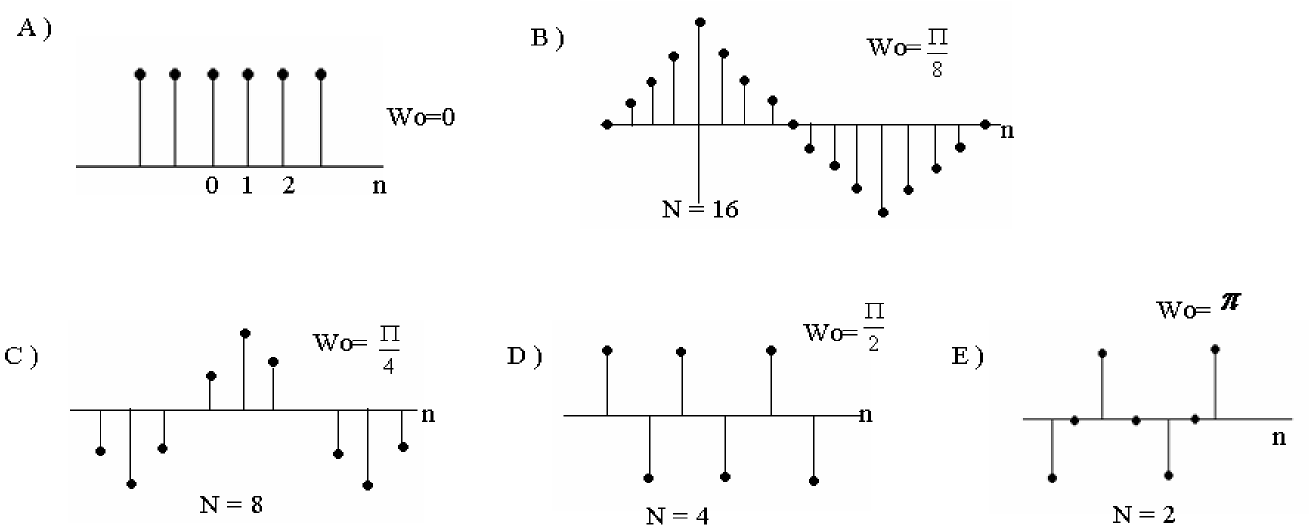
f_o = GCD (f_1, f_2) & T = LCM (T_1, T_2) ------- For Analog/digital signal

[Complex exponential and sinusoidal sequences are not necessarily periodic in 'n' with period ($\frac{2\Pi}{Wo}$) and depending on Wo, may not be periodic at all]

N = fundamental period of a periodic sinusoidal.

3. The highest rate of oscillations in a discrete time sinusoid is obtained when w = π or -π

$$X(n) = \text{Cos}(W_o n)$$



Discrete-time sinusoidal signals with frequencies that are separated by an integral multiple of 2π are Identical.

4. $-\frac{Fs}{2} \leq F \leq \frac{Fs}{2}$

$-\pi Fs \leq 2\pi F \leq \pi Fs$

$-\frac{\Pi}{Ts} \leq \Omega \leq \frac{\Pi}{Ts}$

$-\pi \leq \Omega Ts \leq \pi$

Therefore $-\pi \leq w \leq \pi$

5. Increasing the frequency of a discrete- time sinusoid does not necessarily decrease the period of the signal.

$x_1(n) = \text{Cos}\left(\frac{\Pi n}{4}\right)$ N=8

$x_2(n) = \text{Cos}\left(\frac{3\Pi n}{8}\right)$ N=16 3/8 > 1/4

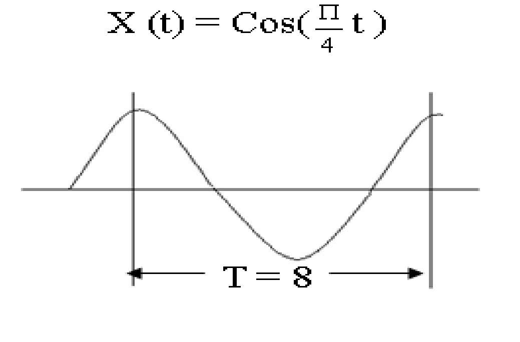
$2\pi f = 3\pi/8$

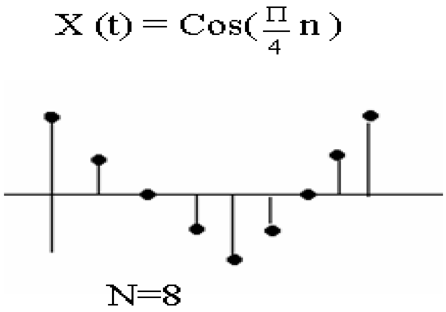
$\Rightarrow f = \frac{3}{16}$

6. If analog signal frequency = F = $\frac{1}{Ts}$ samples/Sec = Hz then digital frequency f = 1

$$W = \Omega\, T_s$$

$$2\pi f = 2\pi F\, T_s \qquad \Rightarrow \qquad f = 1$$

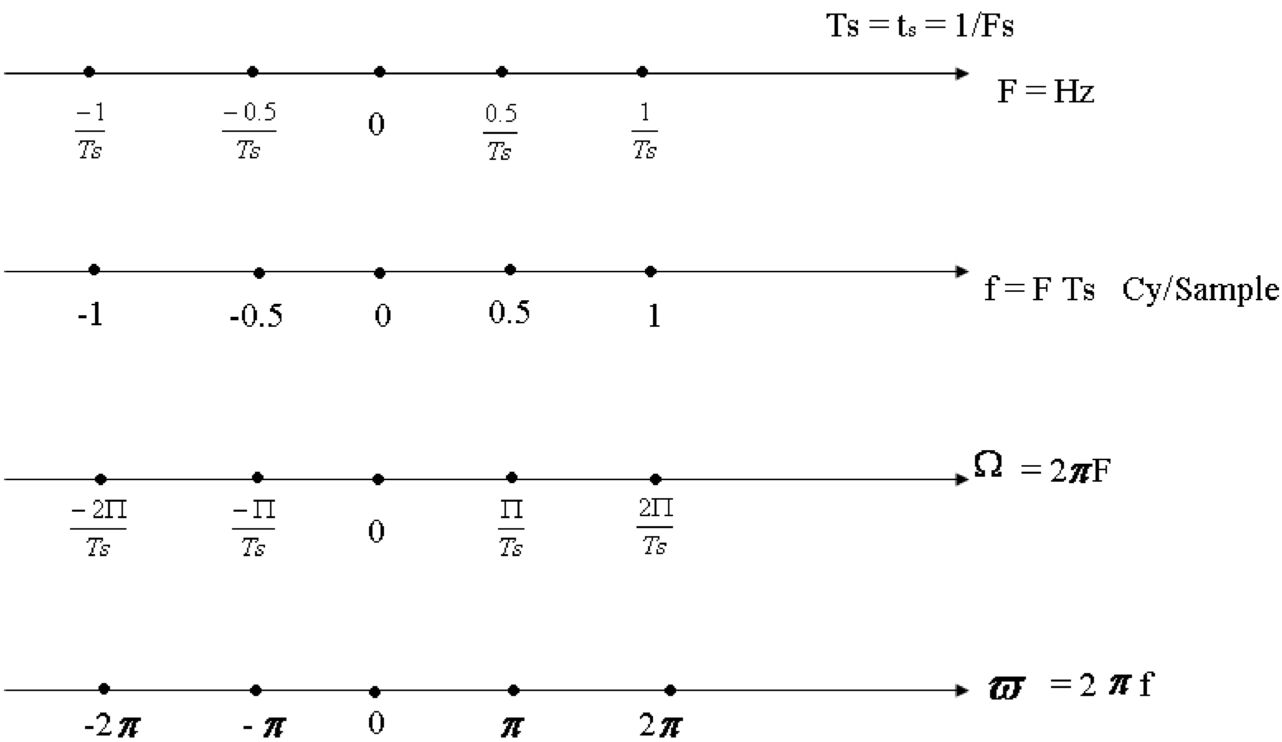




$2\pi F = \dfrac{\Pi}{4}$; $\qquad\qquad 2\pi f = \pi/4$

$F = \dfrac{1}{8}$; $T = 8$; $\qquad\qquad f = \dfrac{1}{8} \qquad N=8$

7. Discrete-time sinusoids are always periodic in frequency.



Q. The signal x (t) = 2 Cos (40πt) + Sin (60πt) is sampled at 75Hz. What is the common period of the sampled signal x (n), and how many full periods of x (t) does it take to obtain one period of x(n)?

F_1 = 20Hz F_2 = 30Hz

$f_1 = \frac{20}{75} = \frac{4}{15} = \frac{K1}{N1}$ $f_2 = \frac{30}{75} = \frac{2}{5} = \frac{K2}{N2}$

The common period is thus N=LCM (N_1, N2) = LCM (15, 5) = 15

The fundamental frequency F_o of x (t) is GCD (20, 30) = 10Hz

And fundamental period T = $\frac{1}{Fo} = 0.1s$

Since N=15

1sample ---------- $\frac{1}{75}$sec

15 sample ----------- ? => $\frac{15}{75} = 0.2S$

$\therefore$ So it takes two full periods of x (t) to obtain one period of x (n) or GCD (K_1, K_2) = GCD (4, 2) = 2

Frequency Domain Representation of discrete-time signals and systems

For LTI systems we know that a representation of the input sequence as a weighted sum of delayed impulses leads to a representation of the output as a weighted sum of delayed responses.

Let x (n) = e^{jwn}

y (n) = h (n) $*$ x (n)

$$= \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{k=-\infty}^{\infty} h(k)\, e^{jw\,(n-k)}$$

$$= e^{jwn} \sum_{k=-\infty}^{\infty} h(k)\, e^{-jwk}$$

Let H (e^{jw}) = $\sum_{k=-\infty}^{\infty} h(k)\, e^{-jwk}$ is the frequency domain representation of the system.

$\therefore$ y (n) = H (e^{jw}) e^{jwn} e^{jwn} = eigen function of the system.

H (e^{jw}) = eigen value

Q. Find the frequency response of 1^{st} order system y (n) = x (n) + a y (n-1) (a<1)

Let x (n) = e^{jwn}

y_p (n) = C e^{jwn}

C e^{jwn} = e^{jwn} + a C $e^{jw(n-1)}$

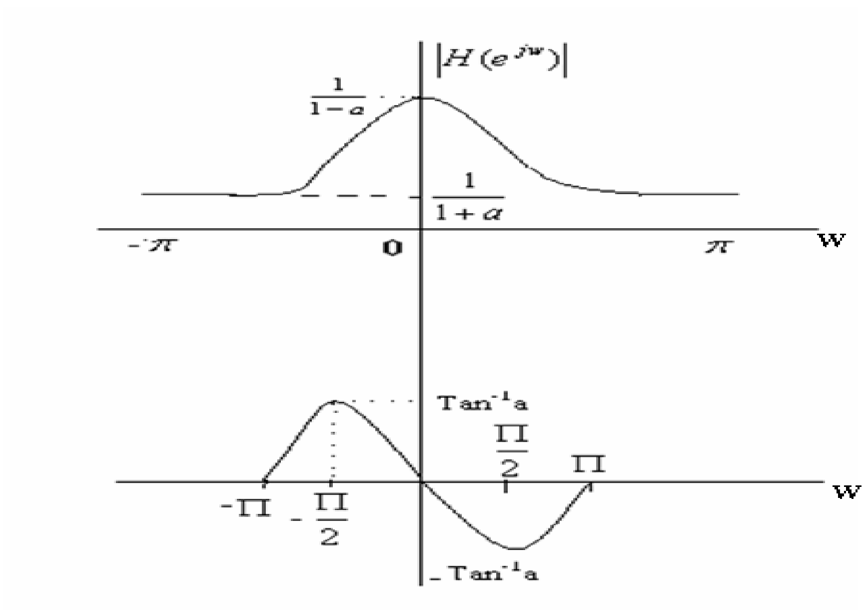
C e^{jwn} [1-ae^{-jw}] = e^{jwn}

$$C = \frac{1}{[1-ae^{-jw}]}$$

Therefore $H(e^{jw}) = \frac{1}{[1-ae^{-jw}]} = \frac{1}{1-a(\cos w - j\sin w)}$

$$\left|H(e^{jw})\right| = \frac{1}{\sqrt{1-2a\cos w + a^2}}$$

$$\angle H(e^{jw}) = -Tan^{-1}\left(\frac{aSinw}{1-aCosw}\right)$$



Q. Frequency response of 2nd order system y(n) = x(n) - $\frac{1}{2}y(n-2)$

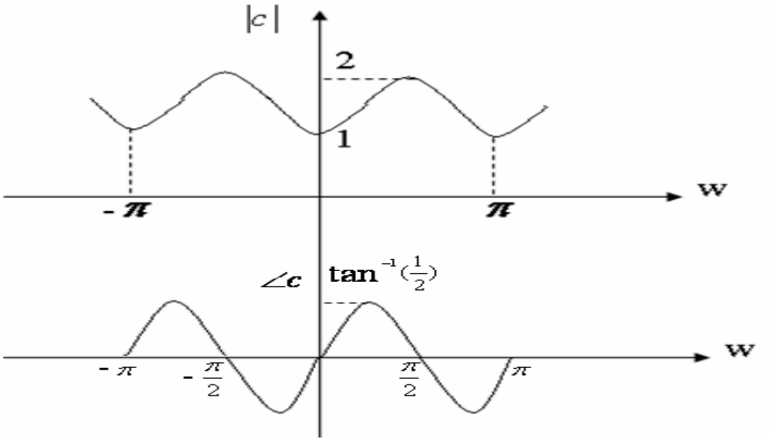
x (n) = e^{jwn}

$y_p(n) = ce^{jwn}$

c$e^{jwn} = e^{jwn}$ - $\frac{1}{2}ce^{jw(n-2)}$

$$c\,e^{jwn}\left(1+\frac{1}{2}e^{-2jw}\right) = e^{jwn} \qquad c = \frac{1}{1+\frac{1}{2}e^{-2jw}} \qquad |c| = \frac{\sqrt{20+16Cos2w}}{5+4Cos2w}$$

$$\angle c = \tan^{-1}\left(\frac{Sin2w}{2+Cos2w}\right)$$



2. POWER, ENERGY and CONVOLUTION

Continuous Time $\Omega_o t = \Omega_o n T_s = w_o n$	Discrete Time
Periodic f (t) = $\sum_{k=-\infty}^{\infty} c_k e^{jk\Omega o t}$ Non periodic $C_k = \frac{1}{T}\int_0^T f(t)e^{-jK\Omega o t}dt$ $\frac{1}{NTs}\sum x(n)e^{-jK\frac{2\pi n Ts}{T}}$ T = N Ts t = n Ts : dt = Ts	Periodic $x_p(n) = \sum_{k=0}^{N-1} c_k e^{jK\frac{2\pi}{N}n}$ DTFS Periodic C_k = $\frac{1}{N}\sum_{n=0}^{N-1} x_p(n)e^{-j\frac{2\pi}{N}nK}$ k=0 to N-1
Non-Periodic f(t) = $\frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{j\Omega t}d\Omega$ Non-Periodic F(w) = $\int_{-\infty}^{\infty} f(t)e^{-j\Omega t}dt$	Non – Periodic x(n) = $\frac{1}{2\pi}\int_0^{2\pi} X(w)e^{jwn}dw$ Periodic X(w) = $\sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$ X(w) = FT of DTS

2.1 Energy and Power

E = $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} x(n)x^*(n) = \sum_{n=-\infty}^{\infty} x(n) \frac{1}{2\pi} \int_0^{2\pi} X^*(w)e^{-jwn} dw$

=

$\frac{1}{2\pi} \int_0^{2\pi} X^*(w) \left[\sum_{n=-\infty}^{\infty} x(n)e^{-jwn} \right] dw$ =

$\frac{1}{2\pi} \int_0^{2\pi} X^*(w) X(w) dw$

= $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(w)|^2 dw$

Therefore: E = $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(w)|^2 dw$ -------- Parsval's Theorem

P = $\underset{N \to \infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$ for non periodic signal

= $\frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$ for periodic Signal

= $\frac{1}{N} \sum_{n=0}^{N-1} x(n)x^*(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \sum_{k=0}^{N-1} C_k{}^* e^{-j\frac{2\pi}{N}nk}$

= $\sum_{k=0}^{N-1} C_k{}^* \left[\frac{1}{N} \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \right]$

Therefore P = $\sum_{k=0}^{N-1} |C_k|^2$ E = N $\sum_{k=0}^{N-1} |Ck|^2$

Ex: Unit step

P = $\underset{N\to\infty}{Lt}\frac{1}{2N+1}\sum_{n=0}^{N}u^2(n)$

= $\underset{N\to\infty}{Lt}\frac{N+1}{2N+1}=\frac{1}{2}$ Power Signal

E = ∞

Ex: x (n) = Ae^{jwon}

P = $\underset{N\to\infty}{Lt}\frac{1}{2N+1}\sum_{n=-N}^{N}\left|Ae^{jwon}\right|^2$

= $\underset{N\to\infty}{Lt}\frac{1}{2N+1}A^2[1+1+........]$

= $\underset{N\to\infty}{Lt}\frac{A^2(2N+1)}{2N+1}=A^2$ it is Power Signal and E = ∞

Ex: x (n) = n u(n) neither energy nor power signal

Ex: x (n) = 3 $(0.5)^n$ $n\geq 0$

E = $\sum_{n=-\infty}^{\infty}x^2(n)=\sum_{n=0}^{\infty}9(0.25)^n=\frac{9}{1-0.25}=12J$ note: [$\sum_{n=0}^{\infty}\alpha^n=\frac{1}{1-\alpha}$]

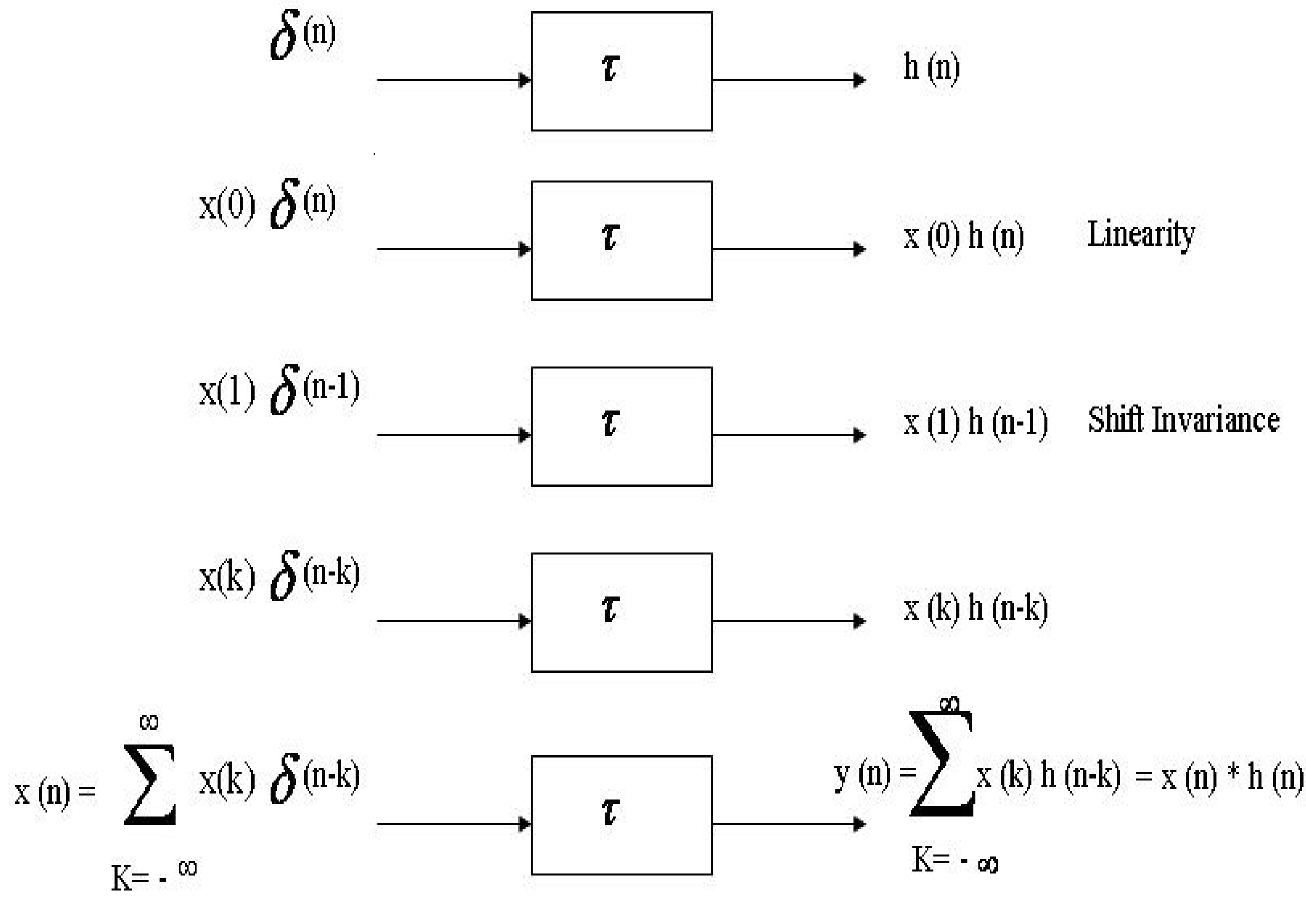
Ex: x (n) = 6 Cos $\frac{2\pi n}{4}$ whose period is N=4 x (n) = { $\underset{\uparrow}{6},0,-6,0$ }

P = $\frac{1}{4}\sum_{n=0}^{3}x^2(n)=\frac{1}{4}[36+36]=18W$

Ex: x (n) = 6 $e^{j\frac{2\pi n}{4}}$ whose period is N = 4

P = $\frac{1}{4}\sum_{n=0}^{3}|x(n)|^2 = \frac{1}{4}[36+36+36+36] = 36 Watts$

2.2 DISCRETE CONVOLUTION



∴ It is a method of finding zero input response of linear Time Invariant system.

Ex: x(n) = u(n)

h(n) = u(n)

y(n) = $\sum_{k=-\infty}^{\infty} u(k)u(n-k)$

u(k) = 0 k<0

$u(n-k) = 0 \quad k>n$

$$\therefore \sum_{k=0}^{n} u(k)u(n-k) = \sum_{k=0}^{n} 1 = (n+1)\,u(n) = r(n+1)$$

Q. $x(n) = a^n u(n)$ and $h(n) = a^n u(n)$ $a<1$ find $y(n)$

$$y(n) = \sum_{k=0}^{n} a^k a^{n-k} = a^n (n+1)\,u(n)$$

Q. $x(n) = u(n)$ and $h(n) = \alpha^n u(n)$ $\alpha<1$ find $y(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} \alpha^k\, u(k)\, u(n-k) = \sum_{k=0}^{n} \alpha^k = (1-\alpha^{n+1}) / (1-\alpha)$$

The convolution of the left sided signals is also left sided and the convolution of two right sided also right sided.

Q. $x(n) = \text{rect}\left(\frac{n}{2N}\right) = 1 \qquad |n| \le N$

$\qquad = 0 \qquad$ else where

$h(n) = \text{rect}\left(\frac{n}{2N}\right)$

$y(n) \quad = x(n) * h(n)$

$\qquad = [u(n+N) - u(n-N-1)] * [u(n+N) - u(n-N-1)]$

$= u(n+N) * [u(n+N) - u(n-N-1)] - u(n-N-1) * [u(n+N) - u(n-N-1)]$

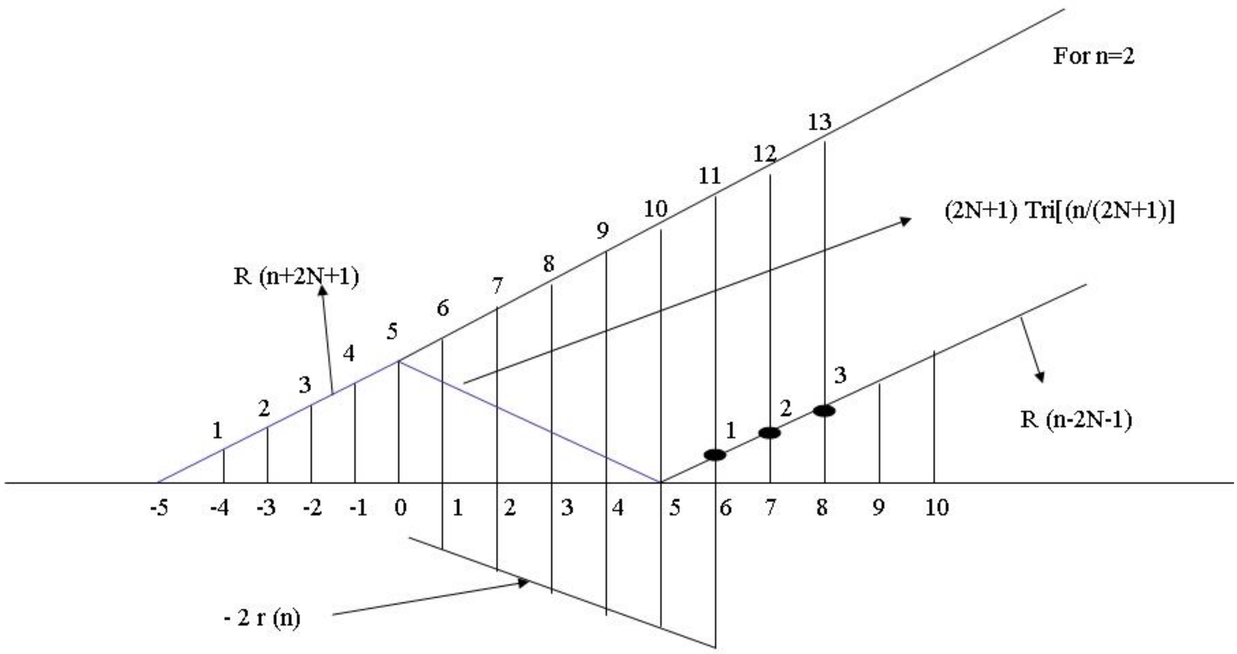
$= u(n+N) * u(n+N) - 2\,u(n+N) * u(n-N-1)] + u(n-N-1) * u(n-N-1)$

$= r(n+2N+1) - 2r(n) + r(n-2N-1)$

$= (2N+1)\,\text{Tri}\left(\frac{n}{2N+1}\right)$

$\text{Tri}\left(\frac{n}{N}\right) = 1 - \frac{|n|}{N}$ for $|n| \le N$

$\qquad = 0 \qquad$ elsewhere.



Q. x(n) = {2,-1,3}

h(n) = { 1,2,2,3} Graphically $\rightarrow$ Fold-shift-multiply-sum

y(n) =

	1	2	2	3
2	2	4	4	6
-1	-1	-2	-2	-3
3	3	6	6	9

y(n) = { 2,3,5,10,3,9}

Q. x(n) = {4, $\underset{\uparrow}{1}$,3} h(n) = { 2,5, $\underset{\uparrow}{0}$,4}

	2	5	0	4
4	8	20	0	16
1	2	5	0	4
3	6	15	0	12

y(n) = { 8,22,11,31,4,12 } Note that convolution starts at n=-3

↑

Q)

h(n):	2	5	0	4		
x(n):	4	1	3			
	8	20	0	16		
		2	5	0	4	
			6	15	0	12
y(n):	8	22	11	$\underset{\uparrow}{31}$	4	12

Q. Convolution by sliding step method: h(n) = $\underset{\uparrow}{2}$, 5, 0, 4 ; x(n)= $\underset{\uparrow}{4}$, 1, 3

i) 2 5 0 4

3 1 4

y(0) = 8

ii) 2 5 0 4

3 1 4

2 20 y(1) = 2+20 = 22

iii) 2 5 0 4

3 1 4

6 5 0 y(2) = 11

iv) 2 5 0 4

3 1 4

15 0 16 y(3) = 31

v) 2 5 0 4

3 1 4

Vi) 2 5 0 4

3 1 4

0 4 y(4) = 4 12 y(5) = 12

If we insert zeros between adjacent samples of each signal to be convolved, their convolution corresponding to the original convolution sequence with zeros inserted between its adjacent samples.

Q. $h(n) = \underset{\uparrow}{2}, 5, 0, 4$; $x(n) = \underset{\uparrow}{4}, 1, 3$ $X(z) = 2z^3+5z^2+4$; $X(z) = 4z^2+z+3$

Their product $Y(z) = 8z^5+22z^4+11z^3+31z^2+4z+12$

$y(n) = \underset{\uparrow}{8},22,11,31,4,12$

$h(n) = \underset{\uparrow}{2}, 0, 5, 0, 0, 0, 4$; $x(n) = 4, 0, 1, 0, 3$

$H(z) = 2z^6+5z^4+ 4$; $X(z) = 4z^4+z^2+3$

$Y(z) = 8z^{10}+22z^6+31z^4+4z^2+12$ $y(n) = \{ 8,0,22,0,11,0,31,0,4,0,12\}$

Q. Compute the linear convolution of h(n)={1,2,1} and x(n) = { 1, -1, 2, 1 ,2, -1, 1, 3, 1} using overlap-add and overlap-save method.

h (n):	1	2	1						
x (n):	1	-1	2	1	2	-1	1	3	1
$x_1(n)$:	1	-1	2						
$x_2(n)$:				1	2	-1			
$x_3(n)$:							1	3	1

$y_1(n) = (h(n) * x_1(n))$ 1 1 1 3 2

$y_2(n) =$ 1 4 4 0 -1

$y_3(n) =$ 1 5 8 5 1

$y(n) = \{ \underset{\uparrow}{1}$ 1 1 4 6 4 1 4 8 5 1 $\}$

OVER LAP and SAVE method

h (n):	1	2	1	0	0	$(N_2=3)$
$x_1(n)$:	1	-1	2	1	2	$(N_3+N_2-1) = 5$

$x_2(n)$: 1 2 -1 1 3

$x_3(n)$: 1 3 1 0

0

$y_1(n)$ = 1 1 1 4 6 5 2

$y_2(n)$ = 1 4 4 1 4 7 3

$y_3(n)$ = 1 5 8 5

1

$y(n)$ = { 1 1 1 4 6 4 1 4 8 5

1}

3. DISCTRETE FOURIER SERIES

Q. Determine the spectra of the signals

a. x(n) = Cos $\sqrt{2}\,\pi$ n

$w_o = \sqrt{2}\,\pi$

$f_o = \frac{1}{\sqrt{2}}$ is not rational number

$\therefore$ Signal is not periodic.

Its spectra content consists of the single frequency

b. x (n) = Cos $\frac{\pi}{3}n$ after expansion x(n)={ 1,0.5,-0.5,-1,-0.5,0.5}

$f_o = \frac{1}{6}$ N=6

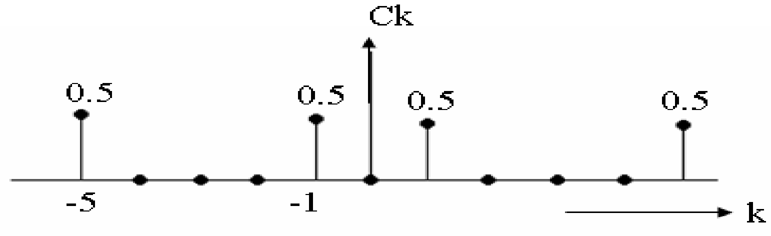
$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)e^{-j\frac{2\pi}{6}nk}$ k=0 to 5

$$C_k = \frac{1}{6}\left[x(0)+x(1)e^{-j\frac{\pi}{3}k}+x(2)e^{-j\frac{2\pi}{3}k}+x(3)e^{-j\pi k}+x(4)e^{-j\frac{4\pi}{3}k}+x(5)e^{-j\frac{5\pi}{3}k}\right]$$

For k=0 Co = $\frac{1}{6}\left[x(0)+x(1)+x(2)+x(3)+x(4)+x(5)\right]$ = 0

Similarly

K=1 $C_1 = 0.5$, $C_2 = 0 = C_3 = C_4$, $C_5 = 0.5$



Or

$$x(n) = \text{Cos}\,\frac{\pi}{3}n = \frac{1}{2}e^{j\frac{2\pi}{6}n} + \frac{1}{2}e^{-j\frac{2\pi}{6}n} = \sum_{k=0}^{5} C_k e^{j\frac{2\pi}{6}kn}$$

$$= C_o + C_1\, e^{j\frac{2\pi}{6}n} + C_2 e^{j\frac{4\pi}{6}n} + C_3\, e^{j\frac{6\pi}{6}n} + C_4\, e^{j\frac{8\pi}{6}n} + C_5\, e^{j\frac{10\pi}{6}n}$$

By comparison $C_1 = \frac{1}{2}$

Since $e^{-j\frac{2\pi}{6}n} = e^{j2\pi\left(\frac{5-6}{6}\right)n} = e^{j\frac{10\pi n}{6}}$

$$\therefore C_5 = \frac{1}{2}$$

c. x (n) = {1,1,0,0}

$$C_k = \frac{1}{4}\sum_{n=0}^{3} x(n)e^{-j2\pi\frac{nk}{4}} \qquad k=0,\ 1,\ 2,\ 3$$

$$= \frac{1}{4}\left[1 + 1e^{-j\frac{2\pi k}{2}}\right]$$

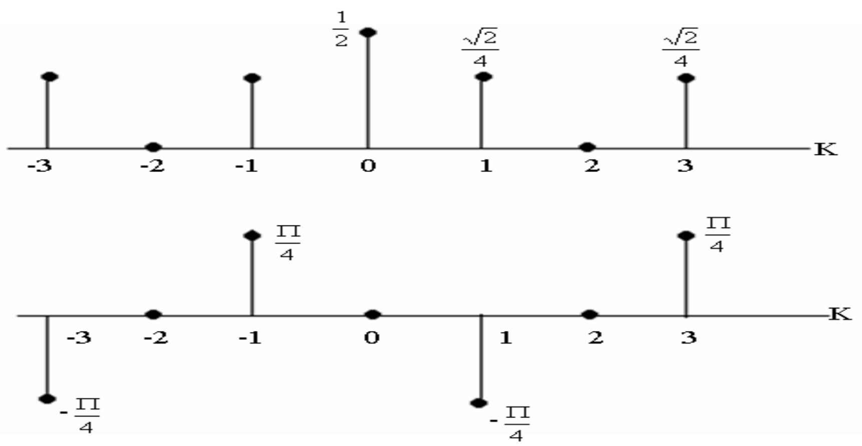
$$C_0 = \frac{1}{2}\ ; \qquad C_1 = \frac{1}{4}(1-j); \qquad C_2 = 0\ ; \quad C_3 = \frac{1}{4}(1+j)$$

$|C_o| = \frac{1}{2}$ & $C_0 = 0$

$|C_1| = \frac{\sqrt{2}}{4}$ & $C_1 = \frac{-\pi}{4}$

$|C_2| = 0$ & C_2 undefined

$|C_3| = \frac{\sqrt{2}}{4}$ & $C_3 = \frac{\pi}{4}$



3.1 PROPERTIES OF DFS

1. Linearity

DFS $\left[\tilde{x}_1(n)\right] = C_{k1}$

DFS $\left[\tilde{x}_2(n)\right] = C_{k2}$

DFS $\left[a\tilde{x}_1(n) + b\tilde{x}_2(n)\right] = aC_{k1} + bC_{k2}$

2. Time Shifting

DFS $\left[\tilde{x}(n-m)\right] = e^{-j\frac{2\pi mk}{N}} C_k$

3. Symmetry

DFS $\left[\tilde{x}^*(n)\right] = C^*_{-k}$ $\qquad$ $C_k =$

$$\frac{1}{N}\sum_{n=0}^{N-1}\tilde{x}(n)e^{-j\frac{2\pi nk}{N}}$$

DFS $\left[\tilde{x}^*(-n)\right] = C^*_k$ $\qquad$ $\tilde{x}(n) = \sum_{k=0}^{N-1} C_k e^{j2\frac{\pi}{N}nk}$

DFS $\left[\mathrm{Re}\left[\tilde{x}(n)\right]\right] = DFS\left[\frac{\tilde{x}(n) + \tilde{x}^*(n)}{2}\right] = \frac{1}{2}\left[C_k + C^*_{-k}\right] = C_{ke}$

DFS

$$\left[j\,\mathrm{Im}\left[\tilde{x}(n)\right]\right] = DFS\left[\frac{\tilde{x}(n) - \tilde{x}^*(n)}{2}\right] = \frac{1}{2}\left[C_k - C^*_{-k}\right] = C_{ko}$$

If $\tilde{x}(n)$ is real then

$$\tilde{x}_e(n) = \left[\frac{\tilde{x}(n) + \tilde{x}^*(-n)}{2}\right]$$

$$\tilde{x}_o(n) = \left[\frac{\tilde{x}(n) - \tilde{x}^*(-n)}{2}\right]$$

$$\text{DFS}\left[\tilde{x}_e(n)\right] = \frac{1}{2}\left[C_k + C^*{}_k\right] = \text{Re}\left[C_k\right]$$

$$\text{DFS}\left[\tilde{x}_o(n)\right] = \frac{1}{2}\left[C_k - C^*{}_k\right] = j\,\text{Im}\left[C_k\right]$$

<u>Periodic Convolution</u>

$$\text{DFS}\quad\left[\sum_{m=0}^{N-1}\tilde{x}_1(m)\tilde{x}_2(n-m)\right] = C_{k1}C_{k2}$$

If x(n) is real

$$C_k = C^*{}_{-k}$$

$$\text{Re}[C_k] = \text{Re}[C_{-k}]$$

$$\text{Im}[C_k] = -\text{Im}[C_{-k}]$$

$$|C_k| = |C_{-k}|$$

$$\angle C_k = -\angle C_{-k}$$

<u>3.2 PROPERTIES OF FT (DTFT)</u>

1. Linearity

y (n) = a x_1 (n) + b x_2 (n)

Y (e^{jw}) = a $X_1(e^{jw})$ + b $X_2(e^{jw})$

2. Periodicity

$$H(e^{j(w+2\pi)}) = H(e^{jw})$$

3. For Complex Sequence

$$h(n) = h_R(n) + j\,h_I(n)$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} [\,h_R(n) + j\,h_I(n)\,][\mathrm{Cos}(wn) - j\,\mathrm{Sin}(wn)]$$

$$\sum_{n=-\infty}^{\infty} [\,h_R(n)\,\mathrm{Cos}(wn) + h_I(n)\mathrm{Sin}(wn) = H_R(e^{jw})$$

$$\sum_{n=-\infty}^{\infty} [\,h_I(n)\,\mathrm{Cos}(wn) - h_R(n)\mathrm{Sin}(wn) = H_I(e^{jw})$$

$$\left|H(e^{jw})\right| = \left|H_R(e^{jw}) + jH_I(e^{jw})\right|$$

$$= \sqrt{H_R^2(e^{jw}) + H_I^2(e^{jw})} = \sqrt{H(e^{jw})H^*(e^{jw})}$$

$$\angle H(e^{jw}) = \tan^{-1}\left[\frac{H_I(e^{jw})}{H_R(e^{jw})}\right]$$

4. For Real Valued Sequence

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h(n)e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} h(n)Cos(wn) - j\sum_{n=-\infty}^{\infty} h(n)Sin(wn)$$

$$= H_R(e^{jw}) - jH_I(e^{jw}) \text{ -------------------- (a)}$$

$$H(e^{-jw}) = \sum_{n=-\infty}^{\infty} h(n)e^{jwn}$$

$$= \sum_{n=-\infty}^{\infty} h(n)Cos(wn) + j\sum_{n=-\infty}^{\infty} h(n)Sin(wn)$$

$= H_R(e^{-jw}) + jH_I(e^{-jw})$ -------------------- (b)

From (a) & (b)

$$H_R(e^{jw}) = H_R(e^{-jw})$$

$$H_I(e^{jw}) = -H_I(e^{-jw})$$

$\therefore$ Real part is even function of w

Imaginary part is odd function of w

$\therefore$ $H(e^{-jw}) = H^*(e^{jw})$

=>

$$\left|H(e^{jw})\right| = \sqrt{H(e^{jw})H^*(e^{jw})} = \sqrt{H^*(e^{-jw})H(e^{-jw})} = \left|H(e^{-jw})\right|$$

$\therefore$ Magnitude response is an even function of frequency

$$\angle H(e^{-jw}) = \tan^{-1}\left[\frac{H_I(e^{-jw})}{H_R(e^{-jw})}\right] = -\tan^{-1}\left[\frac{H_I(e^{jw})}{H_R(e^{jw})}\right] = -\angle H(e^{jw})$$

Phase response is odd function.

5. FT of a delayed Sequence

$$\text{FT}[h(n-k)] = \sum_{n=-\infty}^{\infty} h(n-k)e^{-jwn}$$

Put n-k = m

$$= \sum_{m=-\infty}^{\infty} h(m)e^{-jw(m+k)}$$

$$= e^{-jwk} \sum_{m=-\infty}^{\infty} h(m)e^{-jwm} = H(e^{jw})\, e^{-jwk}$$

6. Time Reversal

x (n) $\rightarrow$ X (w)

x (-n) $\rightarrow$ X (-w)

$$\text{F T [x (-n)]} = \sum_{n=-\infty}^{\infty} x(-n)e^{=jwn}$$

Put $-n = m$

$$\sum_{m=-\infty}^{\infty} x(m)e^{jwm} = X(-w)$$

7. Frequency Shifting

$$x(n)\ e^{jw_o n} \rightarrow X(w-w_o)$$

$$\text{F T } [x(n)\ e^{jw_o n}] = \sum_{n=-\infty}^{\infty} x(n)\ e^{jw_o n}\ e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\ e^{-j(w-w_o)n} = X(w-w_o)$$

8. a. Convolution

$$x_1(n) * x_2(n) \rightarrow X_1(w)\, X_2(w)$$

$$\sum_{n=-\infty}^{\infty} [x_1(n) * x_2(n)]\ e^{-jwn} = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} [x_1(k)\, x_2(n-k)]\ e^{-jwn}$$

Put $n-k = m$

$$= \sum_{n=-\infty}^{\infty} x_1(k) \sum_{m=-\infty}^{\infty} [x_2(m)]\ e^{-jw(m+k)}$$

$$= \sum_{n=-\infty}^{\infty} x_1(k)\ e^{-jwk} \sum_{m=-\infty}^{\infty} [x_2(m)]\ e^{-jwm}$$

$$= X_1(w)\, X_2(w)$$

b. $\frac{1}{2\pi}[X_1(w) * X_2(w)] \rightarrow x_1(n)\, x_2(n)$

9. Parsevals Theorem

$$\sum_{n=-\infty}^{\infty} x_1(n)\, x_2^*(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} [X_1(w)\, X_2^*(w)]\, dw$$

$$n\, x(n) \rightarrow j\, \frac{dX(w)}{dw}$$

10.F T of Even Symmetric Sequence

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h(n)\, e^{-jwn}$$

$$= \sum_{n=-\infty}^{-1} h(n)\, e^{-jwn} + h(0) + \sum_{n=1}^{\infty} h(n)\, e^{-jwn}$$

Let n = -m

$$= \sum_{m=1}^{\infty} h(-m)\, e^{jwm} + h(0) + \sum_{n=1}^{\infty} h(n)\, e^{-jwn}$$

Let h (-m) = h (m) for even

Therefore = $h(0) + 2\sum_{n=1}^{\infty} h(n) \cos(wn)$ is a real valued function of frequency

$\angle\theta = 0$; $H(e^{jw}) > 0$

$\angle\theta = \pm\pi$; $H(e^{jw}) < 0$

11.F T of Odd Symmetric Sequence

For odd sequence h (0) = 0

$$H(e^{jw}) = \sum_{n=1}^{\infty} h(n)\, [e^{-jwn} - e^{jwn}]$$

$$= -j\, 2\sum_{n=1}^{\infty} h(n) \sin(wn)\; H_I(e^{jw})$$ is a imaginary valued function of freq. and a odd function of w

i.e, $H(e^{-jw}) = -H(e^{jw})$

$$\left|H(e^{jw})\right| = H_I(e^{jw}) \quad \text{for} \quad H_I(e^{jw}) > 0$$

$$= -H_I(e^{jw}) \quad \text{for} \quad H_I(e^{jw}) < 0$$

$$\angle H(e^{jw}) = \frac{\pi}{2} \qquad \text{For w over which} \qquad H_I(e^{jw}) > 0$$

$$= \frac{\pi}{2} \pm \pi \qquad \text{for w over which} \qquad H_I(e^{jw}) < 0$$

12. $x(0) = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} X(w)dw$ Central Co-ordinates

$$X(0) = \sum_{n=-\infty}^{\infty} x(n)$$

13. **Modulation**

$$\text{Cos}(w_o n)\, x(n) \rightarrow \frac{X(w+w_0)}{2} + \frac{X(w-w_0)}{2}$$

3.3 FOURIER TRANSFORM OF DISCRETE TIME SIGNALS

$$X(w) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-jwn}$$

F T exists if $\displaystyle\sum_{n=-\infty}^{\infty} \left|x(n)\right| < \infty$

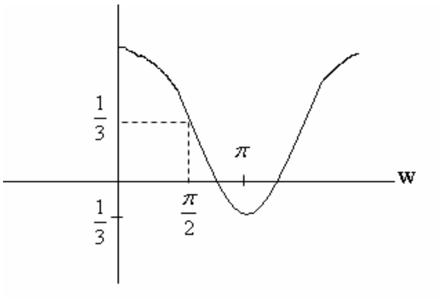
The FT of h (n) is called as Transfer function

Ex: $h(n) = \frac{1}{3}$ for $-1 \le n \le 1$

$= 0$ otherwise

Sol: $H(e^{jw}) = \displaystyle\sum_{n=-1}^{1} \frac{1}{3} e^{-jwn} = \frac{1}{3}\left[e^{jw} + 1 + e^{-jw}\right] = \frac{1}{3}\left[1 + 2Cos(w)\right]$

w	
0	1
$\frac{\pi}{2}$	$\frac{1}{3}$
π	$-\frac{1}{3}$



Ex: $h(n) = a^n u(n)$

$$H(e^{jw}) = \sum_{n=0}^{\infty} a^n e^{-jwn}$$

$$= \sum_{n=0}^{\infty} (ae^{-jw})^n = \frac{1}{1-ae^{-jw}}$$

Q. $x(n) = n\,\alpha^n u(n)$ $\qquad \alpha < 1$

$$n\,\alpha^n u(n) \quad \rightarrow \quad j\frac{d}{dw}\left[\frac{1}{1-\alpha e^{-jw}}\right]$$

$$= \frac{\alpha e^{-jw}}{(1-\alpha e^{-jw})^2}$$

Hint: $\alpha^n u(n) \rightarrow \sum_{n=0}^{\infty} \alpha^n e^{-jwn} = \sum_{n=0}^{\infty} (\alpha e^{-jw})^n = \frac{1}{1-\alpha e^{-jw}}$

Q. $x(n) = \alpha^n \qquad 0 \le n \le N$

Or

$x(n) = \alpha^n [u(n) - u(n-N)]$

$= \alpha^n u(n) - \alpha^N \alpha^{n-N} u(n-N)$ Using Shifting Property

$$X(w) = \frac{1}{1-\alpha e^{-jw}} - \alpha^N \left[\frac{e^{-jwN}}{1-\alpha e^{-jw}}\right]$$

$$= \frac{1-(\alpha e^{-jw})^N}{1-\alpha e^{-jw}} \quad \text{Ans}$$

Q. x(n) = $\alpha^{|n|}$ $|\alpha| < 1$ two sided decaying exponential

x(n) = α^{n} u(n) + α^{-n} u(-n) - $\delta(n)$ using folding property

$$= \frac{1}{1-\alpha e^{-jw}} + \frac{1}{1-\alpha e^{jw}} - 1 = \frac{1-\alpha^2}{1-2\alpha Cos w + \alpha^2}$$

Q. x (n) = u (n) Since u (n) is not absolutely summable

we know that u (t) $\rightarrow \pi\delta(w) + \frac{1}{jw}$

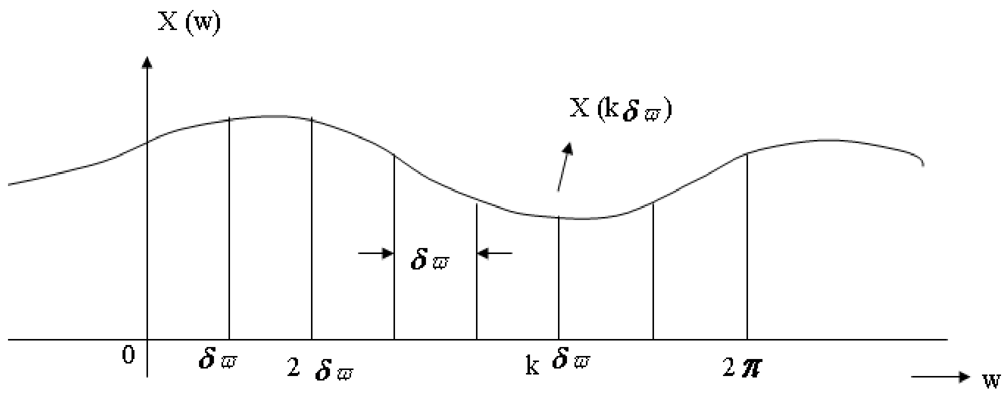
Similarly X (w) = $\frac{1}{1-e^{-jw}} + \pi\delta(w)$

4. DFT (Frequency Domain Sampling)

The Fourier series describes periodic signals by discrete spectra, where as the DTFT describes discrete signals by periodic spectra. These results are a consequence of the fact that sampling on domain induces periodic extension in the other. As a result, signals that are both discrete and periodic in one domain are also periodic and discrete in the other. This is the basis for the formulation of the DFT.

Consider aperiodic discrete time signal x (n) with FT X(w) = $\sum_{n=-\infty}^{\infty} x(n)e^{-jwn}$

Since X (w) is periodic with period 2π, sample X(w) periodically with N equidistance samples with spacing $\delta w = \frac{2\pi}{N}$.



K = 0, 1, 2…..N-1

$$X\left(\frac{2\pi k}{N}\right) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\frac{2\pi}{N}Kn}$$

The summation can be subdivided into an infinite no. of summations, where each sum contains

$$X\left(\frac{2\pi k}{N}\right) = \ldots\ldots\ldots\ldots + \sum_{n=-N}^{-1} x(n)e^{-j\frac{2\pi}{N}Kn} + \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}Kn} +$$

$$\sum_{n=N}^{2N-1} x(n)e^{-j\frac{2\pi}{N}Kn} + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$$

Put n = n-lN

$$= \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}K(n-lN)}$$

$$= \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n-lN) e^{-j\frac{2\pi}{N}Kn}$$

$$X(k) = \sum_{n=0}^{N-1} x_p(n)\ e^{-j\frac{2\pi}{N}Kn}$$

We know that $x_p(n) = \sum_{k=0}^{N-1} C_k\ e^{j\frac{2\pi}{N}Kn}$ n= 0 to N-1

$$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n)\ e^{-j\frac{2\pi}{N}Kn} \qquad k=0 \text{ to } N-1$$

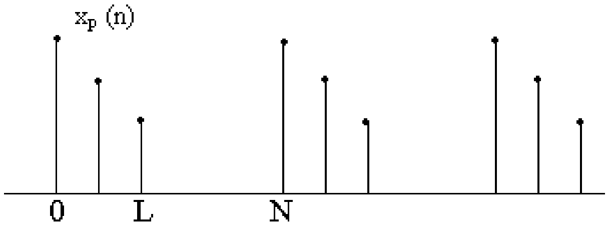
Therefore $C_k = \frac{1}{N} X(k)$ k=0 to N-1

IDFT ------------ $x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\ e^{j\frac{2\pi}{N}Kn}$ n = 0 to N-1

This provides the reconstruction of periodic signal $x_p(n)$ from the samples of spectrum X(w).

The spectrum of aperiodic discrete time signal with finite duration L<N, can be exactly recovered from its samples at frequency $W_k = \frac{2\pi k}{N}$.

Prove: x(n) = x_p (n) $\mathbf{0 \le n \le N-1}$



Using IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\ e^{j\frac{2\pi}{N}Kn}$$

$$X(w) = \sum_{n=0}^{N-1}\left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)\ e^{j\frac{2\pi}{N}Kn}\right] e^{-jwn}$$

$$= \sum_{k=0}^{N-1} X(k)\left[\frac{1}{N}\sum_{n=0}^{N-1} e^{-jn(w-\frac{2\pi}{N}K)}\right]$$

If we define $p(w) = \frac{1}{N}\sum_{n=0}^{N-1} e^{-jwn}$

$$= \frac{1}{N}\left[\frac{1-e^{-jwN}}{1-e^{-jw}}\right] = \frac{Sin\frac{wN}{2}}{NSin\frac{w}{2}}e^{-jw\left(\frac{N-1}{2}\right)}$$

Therefore: $X(w) = \sum_{k=0}^{N-1} X(k)\ P(w-\frac{2\pi k}{N})$

At $w = \frac{2\pi k}{N}$ $\quad P(0) = 1$

And $P(w-\frac{2\pi k}{N}) = 0$ for all other values

$$\therefore X(w) = \sum_{k=0}^{N-1} X(k) = \sum_{k=0}^{N-1} X(\frac{2\pi k}{N})$$

Ex: $\quad x(n) = a^n u(n) \qquad 0<a<1$

The spectrum of this signal is sampled at frequency $W_k=\frac{2\pi k}{N}$. k=0, 1…..N-1, determine reconstructed spectra for a = 0.8 and N = 5 & 50.

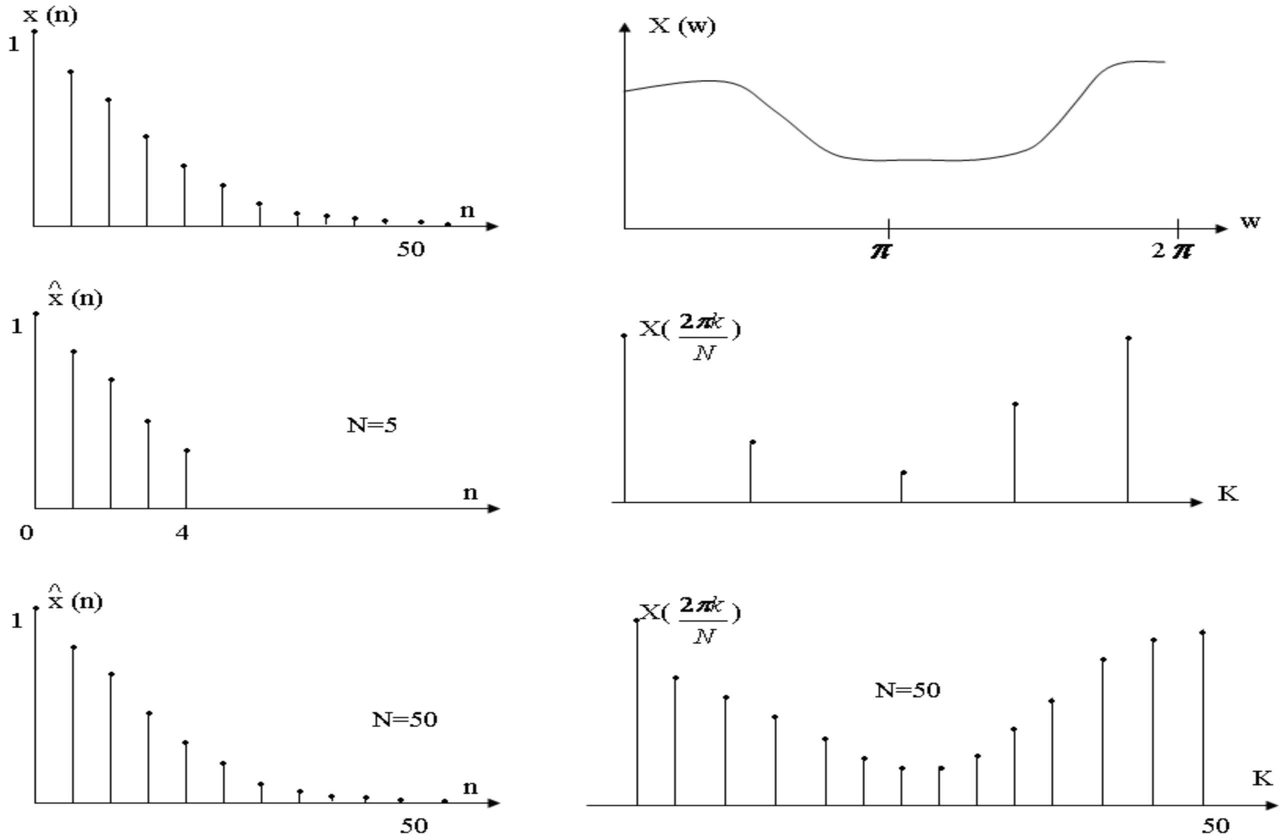
$$X(w) = \frac{1}{1-ae^{-jw}}$$

$$X(w_k) = \frac{1}{1-ae^{-j\frac{2\pi}{N}k}} \qquad k=0, 1, 2\ldots N-1$$

$$x_p(n) = \sum_{l=-\infty}^{\infty} x(n-LN) = \sum_{l=-\infty}^{0} a^{n-lN}$$

$$= a^n \sum_{l=0}^{\infty} a^{lN} = \frac{a^n}{1-a^N} \qquad \mathbf{0 \le n \le N\text{-}1}$$

Aliasing effects are negligible for N=50



If we define aliased finite duration sequence x(n)

$\hat{x}(n) = x_p(n) \qquad 0 \le n \le N\text{-}1$

$\qquad = 0 \qquad$ otherwise

$$\hat{X}(w) = \sum_{n=0}^{N-1} \hat{x}(n) e^{-jwn} \qquad = \sum_{n=0}^{N-1} x_p(n) e^{-jwn}$$

$$= \sum_{n=0}^{N-1} \frac{a^n}{1-a^N} e^{-jwn} \qquad = \frac{1}{1-a^N} \sum_{n=0}^{N-1} (ae^{-jw})^n$$

$$\hat{X}(w) = \frac{1}{1-a^N} \left[\frac{1-a^N e^{-jwN}}{1-a\ e^{-jw}} \right]$$

$$\hat{X}\left(\frac{2\pi K}{N}\right) = \frac{1}{1-a^N} \left[\frac{1-a^N e^{-j\frac{2\pi k}{N}N}}{1-a\ e^{-j\frac{2\pi K}{N}}} \right]$$

$$= \frac{1}{1-ae^{\frac{-j2\pi k}{N}}} = X\left(\frac{2\pi K}{N}\right)$$

$\therefore$ *Although* $\hat{X}(w) \neq X(w), the$ samples at $W_k = \dfrac{2\pi k}{N}$ are identical.

Ex: X (w) = $\dfrac{1}{1-a\ e^{-jw}}$ & X (k) = $\dfrac{1}{1-a\ e^{-j\frac{2\pi}{N}k}}$

Apply IDFT

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\frac{e^{j\frac{2\pi nk}{N}}}{1-ae^{-j\frac{2\pi k}{N}}}\right]$$ using Taylor series expansion

$$= \frac{1}{N}\sum_{k=0}^{N-1}\left[e^{j\frac{2\pi nk}{N}}\right]\sum_{r=0}^{\infty}a^r\ e^{-j\frac{2\pi kr}{N}}$$

$$= \frac{1}{N}\sum_{r=0}^{\infty}a^r\left[\sum_{k=0}^{N-1}e^{j2\pi k\frac{(n-r)}{N}}\right]$$

= 0 except r = n+mN

$$\therefore x(n) = \sum_{m=0}^{\infty}a^{n+mN} = a^n\sum_{m=0}^{\infty}(a^N)^m$$

$$= \frac{a^n}{1-a^N}$$

The result is not equal to x (n), although it approaches x (m) as N becomes ∞.

Ex: x (n) = {0, 1, 2, 3} find X (k) =?

$$X(k) = \sum_{n=0}^{3}x(n)e^{-j\frac{2\pi k}{4}n}$$

$$X(0) = \sum_{n=0}^{3}x(n) = 0+1+2+3 = 6$$

$$X(1) = \sum_{n=0}^{3} x(n)e^{-j\frac{2\pi}{4}n} = -2+2j$$

X (2) = -2

X (3) = -2-2j

4.1 DFT as a linear transformation

Let $W_N = e^{-j\frac{2\pi}{N}}$

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} \qquad k = 0 \text{ to } N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)W_N^{-nk} \qquad n = 0, 1 \ldots N-1$$

Let $x_N = \begin{bmatrix} x(0) \\ x(1) \\ . \\ . \\ x(N-1) \end{bmatrix}$ $\quad X_N = \begin{bmatrix} X(0) \\ X(1) \\ . \\ . \\ X(N-1) \end{bmatrix}$

$$W_N = \begin{bmatrix} 1 & 1 & 1 & \cdots\cdots & 1 \\ 1 & W_N^1 & W_N^2 & \cdots\cdots & W_N^{(N-1)} \\ 1 & W_N^2 & W_N^4 & \cdots\cdots & W_N^{2(N-1)} \\ . & . & . & \cdots\cdots & . \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots\cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

The N point DFT may be expressed in matrix form as

<u>DFT</u> <u>IDFT</u>

$X_N = W_N \; x_N$ $\qquad x_N = \frac{1}{N} W_N^* X_N$

$$\Rightarrow x_N = W_N^{-1} X_N$$

1. $W_N^{K+N} = W_N^K$

$\therefore W_N^{-1} = \frac{1}{N} W_N^*$ 2. $W_N^{K+\frac{N}{2}} = -W_N^K$

Ex: **x (n) = {0, 1, 2, 3}**

<u>DFT</u> $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$X_4 = W_4 x_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

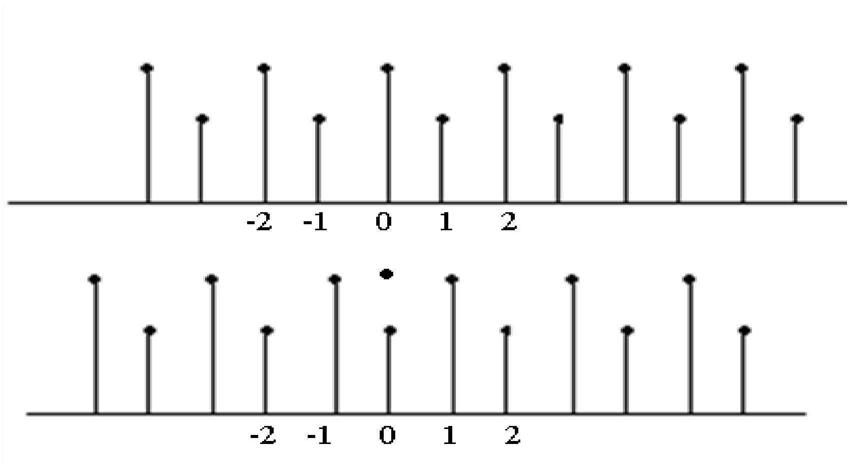
<u>IDFT</u>

$x_4 = \frac{1}{4} W_N^* X_N = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & +j \end{bmatrix} \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$ Ans

Q.

x (n) = $\{\underset{\uparrow}{1}, 0.5\}$

h (n) = $\{\underset{\uparrow}{0.5}, 1\}$



Find y (n) = x (n) $\oplus$ h (n) using frequency domain. Since y (n) is periodic with period 2.

Find 2-point DFT of each sequence.

X (0) = 1.5 H (0) = 1.5

X (1) = 0.5 H (1) = -0.5

Y (K) = X (K) H (K)

Y (0) = 2.25 Y (1) = -0.25

Using IDFT y (0) = 1; y (1) = 1.25

$$\tilde{y}(n) = \tilde{h}(n) \oplus \tilde{x}(n) = \sum_{k=-\infty}^{\infty} \tilde{h}(k)\tilde{x}(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \tilde{x}(k)\tilde{h}(n-k)$$

$$\tilde{y}(0) = \sum_{k=-\infty}^{\infty} \tilde{x}(k)\tilde{h}(-k)$$

$$= \tilde{x}(0)\tilde{h}(0) + \tilde{x}(1)\tilde{h}(-1)$$

= 1 * 0.5 + 0.5 * 1 = 1

$$\tilde{y}(1) = \sum_{k=-\infty}^{\infty} \tilde{x}(k)\tilde{h}(1-k)$$

$= \tilde{x}(0)\tilde{h}(1) + \tilde{x}(1)\tilde{h}(0)$

= 1 * 1 + 0.5 * 0.5 = 1.25

$\tilde{y}(2) = \sum_{k=-\infty}^{\infty} \tilde{x}(k)\tilde{h}(2-k)$

$= \tilde{x}(0)\tilde{h}(2) + \tilde{x}(1)\tilde{h}(1)$

= 1 * 0.5 + 0.5 * 1 = 1

$\tilde{y}(n)$ = {1, 1.25, 1, 1.25…..}

Q. Find Linear Convolution of same problem using DFT

Sol. The linear convolution will produce a 3-sample sequence. To avoid time aliasing we convert the 2-sample input sequence into 3 sample sequence by padding with zero.

For 3- point DFT

X (0) = 1.5 H (0) = 1.5

X (1) = 1+0.5 $e^{-j\frac{2\pi}{3}}$ H (1) = 0.5+ $e^{-j\frac{2\pi}{3}}$

X (2) = 1+0.5 $e^{-j\frac{4\pi}{3}}$ H (2) = 0.5+ $e^{-j\frac{4\pi}{3}}$

Y (K) = H (K) X (K)

Y (0) = 2.25

Y (1) = 0.5 + 1.25 $e^{-j\frac{2\pi}{3}}$ + 0.5 $e^{-j\frac{4\pi}{3}}$

Y (2) = 0.5 + 1.25 $e^{-j\frac{4\pi}{3}}$ + 0.5 $e^{-j\frac{8\pi}{3}}$

Compute IDFT

y(n) = $\frac{1}{3}\sum_{k=0}^{2} Y(k) e^{j\frac{2\pi kn}{3}}$

$y(0) = 0.5$

$y(1) = 1.25$

$y(2) = 0.5$

$y(n) = \{ 0.5, 1.25, 0.5\}$ Ans

4.2 PROPERTIS OF DFT

1) Linearity

If $h(n) = a\, h_1(n) + b\, h_2(n)$

$H(k) = a\, H_1(k) + b\, H_2(k)$

2) Periodicity $H(k) = H(k+N)$

3) $$\tilde{h}(n) = \sum_{m=-\infty}^{\infty} h(n+mN)$$

4) $y(n) = x(n-n_0)$

$$Y(k) = X(k)\, e^{-j\frac{2\pi k n_0}{N}}$$

5) $y(n) = h(n) * x(n)$

$Y(k) = H(k)\, X(k)$

6) $y(n) = h(n)\, x(n)$

$$Y(k) = \frac{1}{N}\left[H(k) \oplus X(k)\right]$$

7) For real valued sequence

$$H_R(k) = \sum_{n=0}^{N-1} h(n) Cos \frac{2\pi kn}{N}$$

$$H_I(k) = -\sum_{n=0}^{N-1} h(n) Sin \frac{2\pi kn}{N}$$

a. Complex conjugate symmetry

$h(n) \rightarrow H(k) = H^*(N-k)$

$h(-n) \rightarrow H(-k) = H^*(k) = H(N-k)$

i. Produces symmetric real frequency components and anti symmetric imaginary frequency components about the $\frac{N}{2}$ DFT

ii. Only frequency components from 0 to $\frac{N}{2}$ need to be computed in order to define the output completely.

b. Real Component is even function

$H_R(k) = H_R(N-k)$

c. Imaginary component odd function

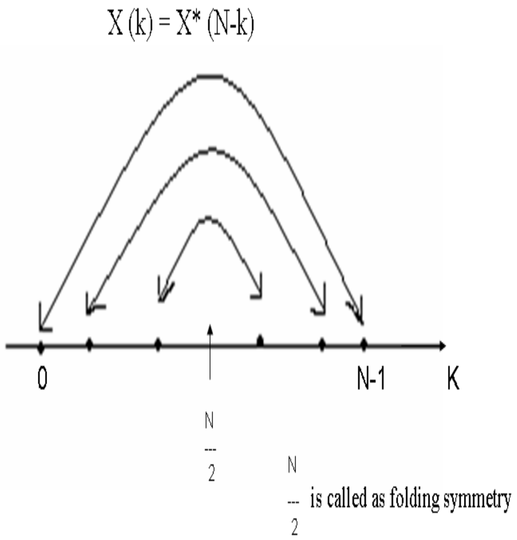
$H_I(k) = -H_I(N-k)$

d. Magnitude function is even function

$$|H(k)| = |H(N-k)|$$

e. Phase function is odd function

$$\angle H(k) = -\angle H(N-k)$$



f. If h(n) = h(-n)

H (k) is purely real

g. If h(n) = -h(-n)

H (k) is purely imaginary

8. For a complex valued sequence

$x^*(n) \leftrightarrow X^*(N-k) = X^*(-k)$

$$\text{DFT}\,[x(n)] = X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$X^*(k) = \sum_{n=0}^{N-1} x^*(n) W_N^{-nk}$$

$$X^*(N-k) = \sum_{n=0}^{N-1} x^*(n) W_N^{nk} = X^*(-k)$$

$$\text{DFT}\,[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) W_N^{nk} = X^*(N-k) \quad \text{proved}$$

Similarly DFT $[x^*(-n)] = X^*(k)$

9.Central Co-ordinates

x (0) = $\frac{1}{N}\sum_{k=0}^{N-1} X(k)$ x $(\frac{N}{2})$ = $\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X(k)$ N=even

X (0) = $\sum_{n=0}^{N-1} x(n)$ X $(\frac{N}{2})$ = $\sum_{n=0}^{N-1}(-1)^n x(n)$ N=even

10. Parseval's Relation

$$N\sum_{n=0}^{N-1}|x(n)|^2 = \sum_{k=0}^{N-1}|X(k)|^2$$

Proof: LHS $N\sum_{n=0}^{N-1} x(n)x^*(n)$

$$= N\sum_{m=0}^{N-1} x(n)\left[\frac{1}{N}\sum_{k=0}^{N-1} X^*(k)W_N^{nk}\right]$$

$$= \sum_{k=0}^{N-1} X^*(k)\left[\sum_{n=0}^{N-1} x(n)W_N^{nk}\right]$$

$$= \sum_{k=0}^{N-1} X^*(k)\,X(k) = \sum_{k=0}^{N-1}|X(k)|^2$$

11.Time Reversal of a sequence

$$x((-n))_N = x(N-n) \leftrightarrow X((-k))_N = X(N-k)$$

Reversing the N-point seq in time is equivalent to reversing the DFT values.

$$\text{DFT } [x(N-n)] = \sum_{n=0}^{N-1} x(N-n)e^{\frac{-j2\pi k}{N}n}$$

Let m=N-n

$$= \sum_{n=0}^{N-1} x(m)e^{\frac{j2\pi k}{N}(N-m)}$$ m=1 to N = 0 to N-1

$$= \sum_{m=0}^{N-1} x(m)e^{\frac{-j2\pi k}{N}m}$$

$$= \sum_{m=0}^{N-1} x(m) e^{\frac{-j2\pi m}{N}(N-k)} = X(N-k)$$

12.Circular Time Shift of a sequence

$$x(n-l)_N \longleftrightarrow X(k) e^{\frac{-j2\pi k}{N} l}$$

$$DFT\left[x(n-l)_N\right] = \sum_{n=0}^{N-1} x(n-l)_N e^{\frac{-j2\pi k}{N} n}$$

$$= \sum_{n=0}^{l-1} x(n-l)_N e^{\frac{-j2\pi k}{N} n} + \sum_{n=l}^{N-1} x(n-l)_N e^{\frac{-j2\pi k}{N} n}$$

$$= \sum_{n=0}^{l-1} x(N+n-l) \; e^{\frac{-j2\pi k}{N} n} + \sum_{n=l}^{N-1} x(N+n-l) e^{\frac{-j2\pi k}{N} n}$$

Put N+n-l = m

$$= \sum_{m=N-l}^{N-1} x(m) e^{\frac{-j2\pi k}{N}(m+l)} + \sum_{m=N}^{2N-1-l} x(m) e^{\frac{-j2\pi k}{N}(m+l)}$$

N to 2N-1-l is shifted to N $\Rightarrow$ 0 to N-1-l

Therefore 0 to N-1 = (0 to N-1-L) to (N-L to N-1)

Therefore $\sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi k}{N}(m+l)}$

$$= \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi k}{N} m} \quad e^{-j\frac{2\pi k}{N} l}$$

$= X(k) \; e^{-j\frac{2\pi k}{N} l}$ RHS

13.Circular Frequency Shift

$$x(n)e^{j\frac{2\pi l}{N}n} \leftrightarrow X(k-l)_N$$

DFT $\left[x(n)e^{j\frac{2\pi l}{N}n} \right] = \sum_{n=0}^{N-1} x(n)e^{j\frac{2\pi l}{N}n} e^{-j\frac{2\pi k}{N}n}$

$= \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi n}{N}(k-l)} = X(k-l)_N$ RHS

14.x(n) $\longleftrightarrow$ X(k)

{x(n), x(n), x(n)…….x(n)} $\Leftrightarrow$ M X ($\frac{k}{m}$)

(m-fold replication)

$x(\frac{n}{m}) \Leftrightarrow \{X(k), X(k),......X(k)\}$ (M- fold replication)

2, 3, 2, 1 $\rightarrow$ 8, -j2, 0, j2

Zero interpolated by M

{2, 3, 2, 1, 2, 3, 2, 1, 2, 3, 2, 1} $\rightarrow$ {24, 0, 0, -j6, 0, 0, 0, 0, 0, j6, 0, 0}

15.Duality

x(n)$\leftrightarrow$X(k)

X(n)$\leftrightarrow$N x(N-k) $0 \le K \le N-1$

x(n) = $\frac{1}{N}\sum_{\lambda=0}^{N-1} X(\lambda)e^{j\frac{2\pi\lambda}{N}n}$

x(N-k) = $\frac{1}{N}\sum_{\lambda=0}^{N-1} X(\lambda)e^{j\frac{2\pi\lambda}{N}(N-k)}$

$= \frac{1}{N}\sum_{\lambda=0}^{N-1} X(\lambda)e^{-j\frac{2\pi\lambda}{N}k}$

N x(N-k) = $\sum_{\lambda=0}^{N-1} X(\lambda) e^{-j\frac{2\pi\lambda}{N}k}$

= $\sum_{n=0}^{N-1} X(n) e^{-j\frac{2\pi k}{N}n}$ = DFT [X(n)] LHS proved

16.Re[x(n)] ↔ $X_{ep}(k)$ $X_{ep}(k)$ = $\frac{1}{2}\left[X((k))_N + X^*((-k))_N\right]$

j Im[x(n)] ↔ $X_{op}(k)$

$x_{ep}(n) \leftrightarrow$ Re[X(k)]

$x_{op}(n) \leftrightarrow$ j Im[X(k)]

$x_{ep}(n) =$ Even part of periodic sequence = $\frac{1}{2}[x(n) + x((-n))_N]$

$x_{op}(n) =$ Odd part of periodic sequence = $\frac{1}{2}[x(n) - x((-n))_N]$

Proof: X(k) = $\sum_{n=0}^{N-1} x(n) W_N^{nk}$

X(N-k) = $\sum_{n=0}^{N-1} x(n) W_N^{-nk} = X((-k))_N$

$X^*(k)$ = $\sum_{n=0}^{N-1} x^*(n) W_N^{-nk}$

$X^*(N-k)$ = $\sum_{n=0}^{N-1} x^*(n) W_N^{nk} = X^*((-k))_N$

$$\frac{X((k))_N + X^*((-k))_N}{2} = \frac{1}{2}\sum_{n=0}^{N-1}\left[x(n) + x^*(n)\right]W_N^{nk}$$

= DFT of [Re[x (n)]]   LHS

17. $\sum_{n=0}^{N-1} x_1(n)x_2^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2^*(k)$

Let y(n) = $x_1(n)x_2^*(n)$

Y(k) = $\frac{1}{N}\left[X_1(k) \oplus X_2^*(-k)\right]$

$= \frac{1}{N}\sum_{l=0}^{N-1}\left[X_1(l)X_2^*(k+l)\right]$

Y(0) = $\frac{1}{N}\sum_{l=0}^{N-1}\left[X_1(l)X_2^*(l)\right]$

Using central co-ordinate theorem

Y(0) = $\sum_{n=0}^{N-1} x_1(n)x_2^*(n)$

Therefore $\sum_{n=0}^{N-1} x_1(n)x_2^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2^*(k)$

QUESTIONS

1 Q. (i) {1,0,0,…….0} (impulse) $\leftrightarrow$ {1,1,1…..1} (constant)

(ii) {1,1,1,……1} (constant) $\leftrightarrow$) {N,0,0,…….0} (impulse)

(iii) $\alpha^n \leftrightarrow \frac{1-\alpha^N}{1-e^{\frac{-j2\pi k}{N}}}$ $\left(\sum_{k=0}^{N-1}\left(\alpha e^{\frac{-j2\pi k}{N}}\right)^n = \frac{1-(\alpha^{\frac{-j2\pi k}{N}})^N}{1-\alpha^{\frac{-j2\pi k}{N}}}\right)$

(iv) Cos $\left(\frac{2\pi nk_o}{N}\right) \leftrightarrow \frac{N}{2}\left[\delta(k-k_o) + \delta(k-(N-k_o))\right]$

(Impulse pair)

Or Cos ($2\pi nf$) = Cos (wn)

Sol. x(n) = $$\frac{e^{\frac{j2\pi nk_o}{N}} + e^{\frac{-j2\pi nk_o)}{N}}}{2}$$

$$= \frac{e^{\frac{j2\pi nk_o}{N}} + e^{\frac{j2\pi n(N-k_o)}{N}}}{2}$$

We know that 1 $\leftrightarrow$ N $\delta(k)$

$$x(n)e^{\frac{j2\pi nKo}{N}} \to X(K-Ko)$$

x(n) $\to$ $$\frac{N}{2}\left[\delta(k-k_o)+\delta(k-(N-k_o)\right]$$

I. Inverse DFT of a constant is a unit sample.

II. DFT of a constant is a unit sample.

2 Q. Find 10 point IDFT of

X(k) = 3 k=0

= 1 $1 \le k \le 9$

Sol. X(k) = 1+2 $\delta(k)$

= 1 + $\frac{1}{5}10\ \delta(k)$

x(n) = $\frac{1}{5}+\delta(n)$ Ans

3 Q. Suppose that we are given a program to find the DFT of a complex-valued sequence x(n). How can this program be used to find the inverse DFT of X(k)?

X(k) = $$\sum_{n=0}^{N-1} x(n)W_N^{nk}$$

x(n) = $\frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-nk}$

N x*(n) = $\sum_{k=0}^{N-1} X^*(k) W_N^{nk}$

$\therefore$

1. Conjugate the DFT coefficients X(k) to produce the sequence X*(k).
2. Use the program to fing DFT of a sequence X*(k).
3. Conjugate the result obtained in step 2 and divide by N.

4 Q. $x_p(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5, 0, 0, 0\}$

(i) $f_p(n) = x_p(n-2) = \{\underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 0\}$

(ii) $g_p(n) = x_p(n+2) = \{\underset{\uparrow}{3}, 4, 5, 0, 0, 0, 1, 2\}$

(iii) $h_p(n) = x_p(-n) = \{1, 0, 0, 0, 5, 4, 3, 2\}$

5 Q. x(n) = {1, 1, 0, 0, 0, 0, 0, 0} n = 0 to 7 Find DFT.

X(k) = $\sum_{n=0}^{1} x(n) e^{\frac{-j2\pi k}{8}n}$ = 1 + $e^{\frac{-j\pi k}{4}}$ k = 0 to 7

X(0) = 1+1 = 2

X(1) = 1+ $e^{\frac{-j\pi}{4}}$ = 1.707 - j 0.707

X(2) = 1+ $e^{\frac{-j\pi}{2}}$ = 1- j

X(3) = 1+ $e^{\frac{-j3\pi}{4}}$ = 0.293 - j 0.707

X(4) = 1-1 = 0

By conjugate symmetry X(k) = X*(N-k) = X*(8-k)

$\therefore$ X(5) = X*(3) = 0.293 + j 0.707

X(6) = X*(2) = 1+j

X(7) = X*(1) = 1.707 + j 0.707

$\therefore$ X(k) = { $\underset{\uparrow}{2}$, 1.707 - j 0.707, 0.293 - j 0.707, 1-j, 0, 1+j, 0.293 + j 0.707, 1.707 + j 0.707 }

6 Q. x(n) = {1, 2, 1, 0} N=4

X(k) = {4, -j2, 0, j2}

(i) y(n) = x(n-2) = {1, 0, 1, 2}

Y(k) = X(k) $e^{\frac{-j2\pi k}{4}(no=2)}$ = 4, j2, 0, -j2

(ii) X(k-1) = {j2, 4, -j2, 0}

IDFT $\Rightarrow$ x(n) $e^{\frac{j2\pi}{N}\text{1n}}$

= x(n) $e^{\frac{j\pi n}{2}}$ = {1, j2, -1, 0}

(iii) g(n) = x(-n) = 1, 0, 1, 2

G(k) = X(-k) = X*(k) = {4, j2, 0, -j2}

(iv) p(n) = x*(n) = {1, 2, 1, 0}

P(k) = X*(-k) = {4, j2, 0, -j2}* = {4, -j2, 0, j2}

(v) h(n) = x(n) x(n)

= {1, 4, 1, 0}

H(k) = $\frac{1}{4}\left[X(k) \oplus X(k)\right]$ = $\frac{1}{4}$ [24, -j16, 0, j16] = {6, -j4, 0, j4}

(vi) c(n) = x(n) $\oplus$ x(n)

= {1, 2, 1, 0} $\oplus$ {1, 2, 1, 0} = {2,4,6,4}

C(k) = X(k)X(k) = {16, -4, 0, -4}

(vii) s(n) = x(n) $\oplus$ x(n) = {1, 4, 6, 4, 1, 0, 0}

S(k) = X(k) X(k) = {16, -2.35- j 10.28, -2.18 + j 1.05, 0.02 + j 0.03, 0.02 - j 0.03, -2.18 - j 1.05, -2.35 + j 10.28}

(viii) $\sum |x(n)|^2 = 1 + 4 + 1 + 0 = 6$

$$\frac{1}{4}\sum\left|X(k)\right|^2=\frac{1}{4}[16+4+4]=6$$

5.FFT

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \qquad 0 \le K \le N-1$$

$$= \sum_{n=0}^{N-1} \{ \mathrm{Re}[x(n)] + j\,\mathrm{Im}[x(n)] \} \{ \mathrm{Re}(W_N^{nk}) + j\,\mathrm{Im}(W_N^{nk}) \}$$

$$= \sum_{n=0}^{N-1} \mathrm{Re}[x(n)]\,\mathrm{Re}(W_N^{nk}) - \sum_{n=0}^{N-1} \mathrm{Im}[x(n)]\,\mathrm{Im}(W_N^{nk}) +$$

$$j\{\sum_{n=0}^{N-1} \mathrm{Im}[x(n)]\,\mathrm{Re}(W_N^{nk}) + \mathrm{Im}(W_N^{nk})\mathrm{Re}[x(n)]\}$$

- Direct evaluation of X(k) requires N^2 complex multiplications and N(N-1) complex additions.
- 4 N^2 real multiplications
- { 4(N-1) + 2} N = N(4N-2) real additions

The direct evaluation of DFT is basically inefficient because it does not use the symmetry & periodicity properties $W_N^{K+\frac{N}{2}} = -W_N^{nk}$ & $W_N^{K+N} = W_N^{nk}$

5.1 DITFFT:

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k}$$

(even) (odd)

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_N^{2nk} + W_N^K \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_N^{2nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_{N/2}^{nk} + W_N^K \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_{N/2}^{nk}$$

= Xe(k) + W_N^K Xo(k)

Although k=0 to N-1, each of the sums are computed only for k=0 to N/2 -1, since Xe(k) & Xo(k) are periodic in k with period N/2

For K $\geq$ N/2 $W_N^{K+\frac{N}{2}} = -W_N^K$

X(k) for K$\geq$ N/2

X(k) = X_e(k-N/2) $-W_N^{K-\frac{N}{2}}$ X_o(k-N/2)

N = 8

x(2n) = x_e(n) ; x(2n+1) = x_o(n)

$x_e(0) = x(0)$ $\quad$ $x_o(0) = x(1)$

$x_e(1) = x(2)$ $\quad$ $x_o(1) = x(3)$

$x_e(2) = x(4)$ $\quad$ $x_o(2) = x(5)$

$x_e(3) = x(6)$ $\quad$ $x_o(3) = x(7)$

X(k) = Xe(k) + $W_8^k Xo(k)$ $\quad$ k = 0 to 3

= Xe(k-4) - $W_8^{k-4} Xo(k-4)$ $\quad$ k = 4 to 7

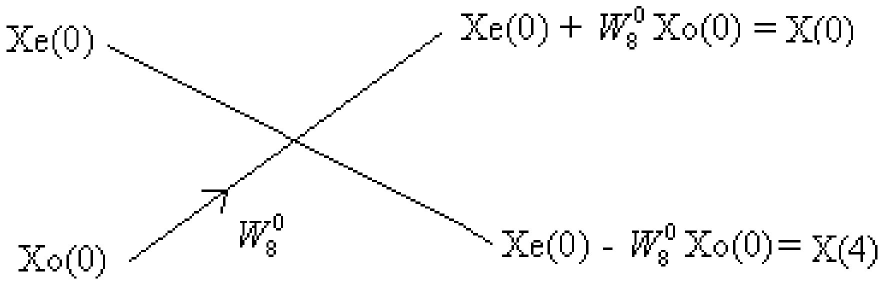
X(0) = Xe(0) + W_8^0 Xo(0) ; X(4) = Xe(0) - W_8^0 Xo(0)

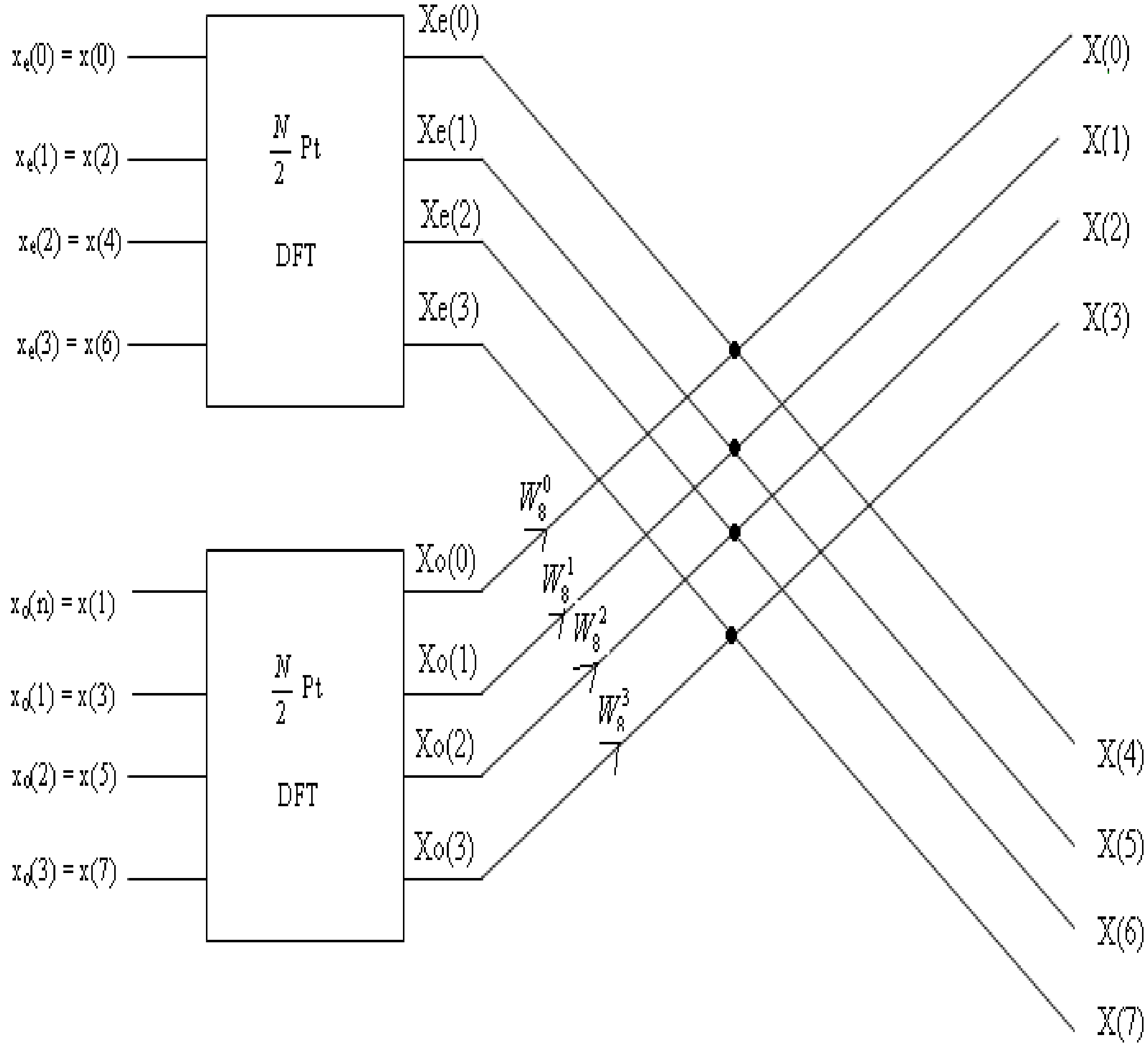
X(1) = Xe(1) + W_8^1 Xo(1) ; X(5) = Xe(1) - W_8^1 Xo(1)

X(2) = Xe(2) + W_8^2 Xo(2) ; X(6) = Xe(2) - W_8^2 Xo(2)

X(3) = Xe(3) + W_8^3 Xo(3) ; X(7) = Xe(3) - W_8^3 Xo(3)

X(0) & X(4) having same i/ps with opposite signs





This $\frac{N}{2}$ pt DFT can be expressed as combination of $\frac{N}{4}$ pt DFT.

$Xe(k) = Xee(k) + W_N^{2k} Xeo(k)$ $\quad$ k = 0 to $\frac{N}{4}$ -1 $\qquad$ (0 to 1)

$= Xee(k-\frac{N}{4}) - W_N^{2(k-\frac{N}{4})} Xeo(k-\frac{N}{4})$ $\quad$ k = $\frac{N}{4}$ to $\frac{N}{2}$ -1 $\qquad$ (2 to 3)

$Xo(k) = Xoe(k) + W_N^{2k} Xoo(k)$ $\quad$ k = 0 to $\frac{N}{4}$ -1

$= Xoe(k-\frac{N}{4}) - W_N^{2(k-\frac{N}{4})} Xoo(k-\frac{N}{4})$ $\quad$ k = $\frac{N}{4}$ to $\frac{N}{2}$ -1

For N=8

$Xe(0) = Xee(0) + W_8^0 Xeo(0)$; $\qquad$ $x_{ee}(0) = x_e(0) = x(0)$

$Xe(1) = Xee(1) + W_8^2\, Xeo(1)$; $x_{ee}(1) = x_e(1) = x(2)$

$Xe(2) = Xee(0) - W_8^0\, Xeo(0)$; $x_{eo}(2) = x_e(2) = x(4)$

$Xe(3) = Xee(1) - W_8^2\, Xeo(1)$; $x_{eo}(3) = x_e(3) = x(6)$

Where Xee(k) is the 2 point DFT of even no. of $x_e(n)$ & Xeo(k) is the 2 point DFT of odd no. of $x_e(n)$

Similarly, the sequence $x_o(n)$ can be divided in to even & odd numbered sequences as

$x_{oe}(0) = x_o(0) = x(1)$

$x_{oe}(1) = x_o(2) = x(5)$

$x_{oo}(0) = x_o(1) = x(3)$

$x_{oo}(1) = x_o(3) = x(7)$

$X_o(0) = Xoe(0) + W_8^0\, Xoo(0)$;

$Xo(1) = Xoe(1) + W_8^2\, Xoo(1)$;

$Xo(2) = Xoe(0) - W_8^0\, Xoo(0)$;

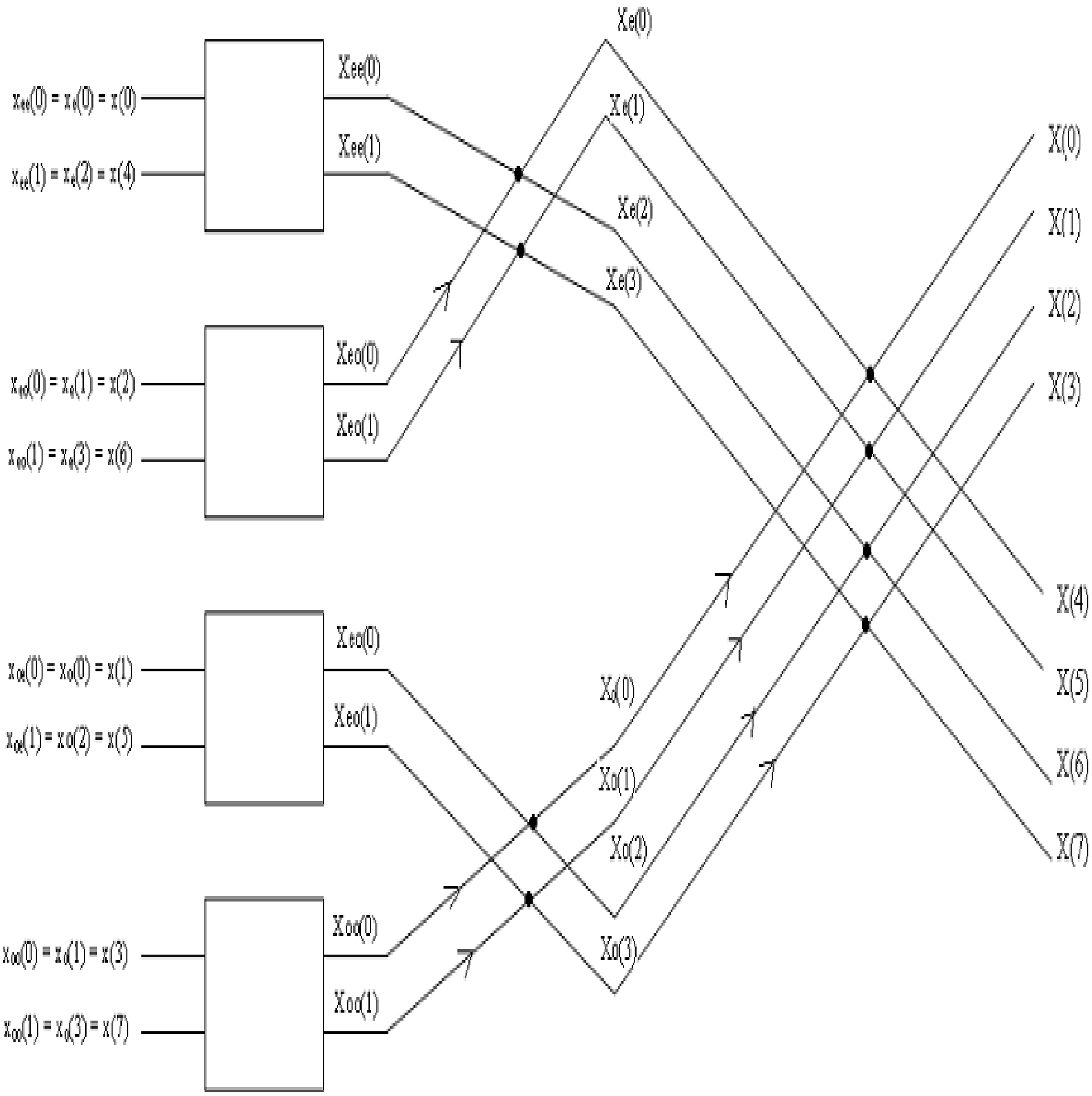
$Xo(3) = Xoe(1) - W_8^2\, Xoo(1)$;

Xoe(k) is the 2-pt DFT of even-numbered of $x_o(n)$

Xoo(k) is the 2-pt DFT of odd-numbered of $x_o(n)$

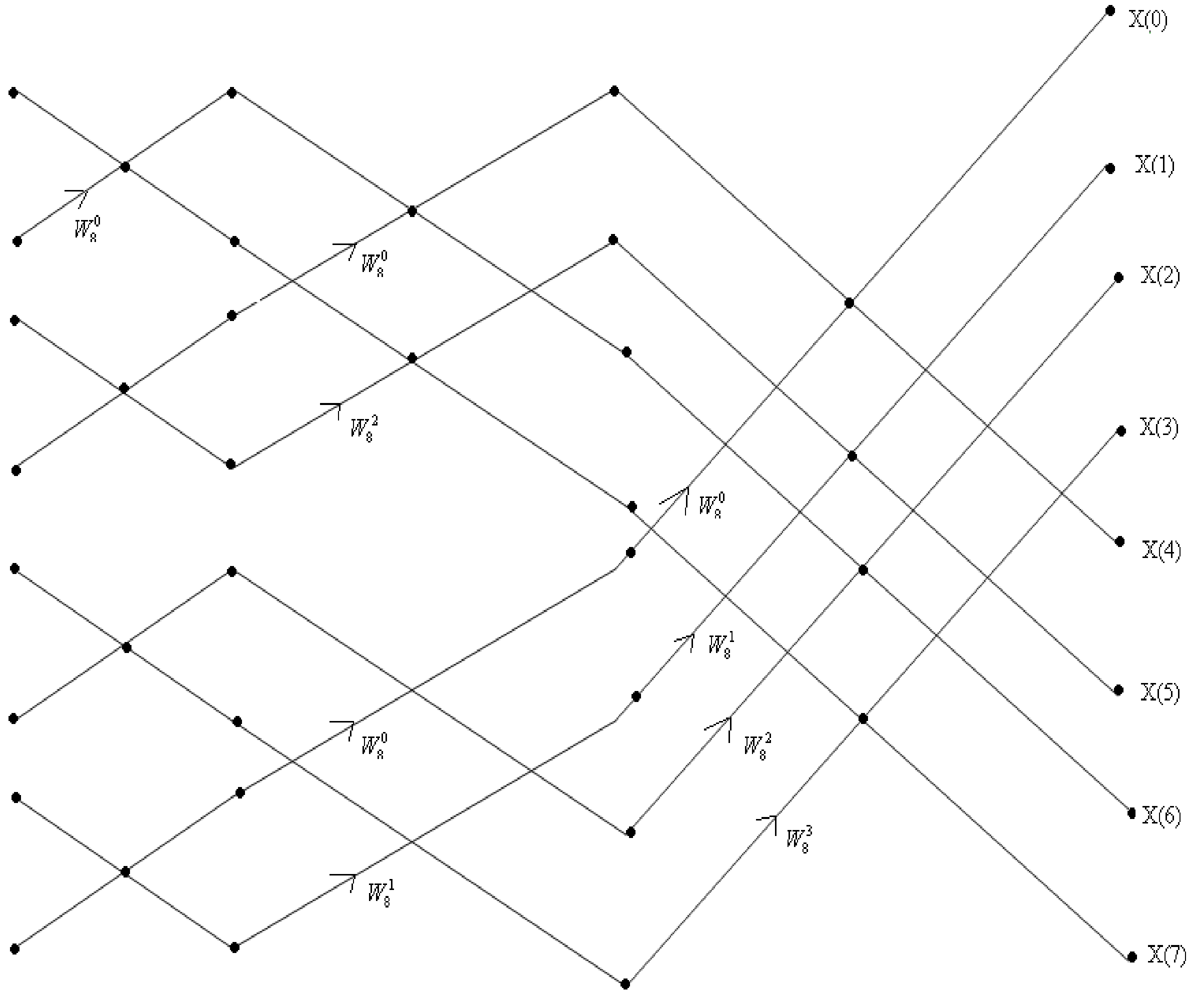
$Xee(0) = x_{ee}(0) + x_{ee}(1) = x_e(0) + x_e(2) = x(0) + x(4)$

$Xee(1) = x_{ee}(0) - x_{ee}(1) = x_e(0) - x_e(2) = x(0) - x(4)$



$Xee(0) = x_{ee}(0) + x_{ee}(1) = x_e(0) + x_e(2) = x(0) + x(4)$

$Xee(1) = x_{ee}(0) - x_{ee}(1) = x_e(0) - x_e(2) = x(0) - x(4)$



No. of Stages	No. of points N	No. of Complex Multiplications		Speed Improvement Factor: $\frac{N^2}{\frac{N}{2}Log_2N}$
		Direct N^2	FFT $\frac{N}{2}Log_2N$	
2	4	16	4	4
3	8	64	12	5.33
4	16	256	32	8
5	32	1024	80	12.8
6	64	4096	192	21.33

For N=8

No of stages given by= $Log_2N = Log_28 = 3$.

No. of 2 i/p sets = $2^{(Log_2N-1)} = 4$

Total No. of Complex additions using DITFFT is $NLog_2N$

= 8 * 3 =24

Each stage no. of butterflies in the stage= 2^{m-q} where q = stage no. and N=2^m

Each butterfly operates on one pair of samples and involves two complex additions and one complex multiplication. No. of butterflies in each stage N/2

DITFFT: (different representation) (u can follow any one) (both representations are correct)

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(2n)W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_e(n)W_{\frac{N}{2}}^{nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_o(n)W_{N/2}^{nk}$$

4 pt DFT Xe(k) + W_N^K Xo(k) k= 0 to N/2 -1 = 0 to 3

Xe(k-$\frac{N}{2}$) - $W_N^{(K-\frac{N}{2})}$ Xo(k-$\frac{N}{2}$) k = N/2 to N-1 = 4 to 7

2 pt DFT Xe(k) = Xee(k) + W_N^{2K} Xeo(k) k = 0 to N/4-1 = 0 to 1

= Xee(k-N/4) - $W_N^{2(k-\frac{N}{4})}$ Xeo(k-N/4) k = N/4 to N/2 -1 = 2 to 3

Xo(k) = Xoe(k) + W_N^{2K} Xoo(k) k = 0 to N/4-1 = 0 to 1

= Xoe(k-N/4) - $W_N^{2(k-\frac{N}{4})}$ Xoo(k-N/4) k = N/4 to N/2 -1 = 2 to 3

$W_8^2 = W_4^1$

N=8

X(0) = Xe(0) + W_8^0 Xo(0) ; X(4) = Xe(0) - W_8^0 Xo(0)

X(1) = Xe(1) + W_8^1 Xo(1) ; X(5) = Xe(1) - W_8^1 Xo(1)

X(2) = Xe(2) + W_8^2 Xo(2) ; X(6) = Xe(2) - W_8^2 Xo(2)

X(3) = Xe(3) + W_8^3 Xo(3) ; X(7) = Xe(3) - W_8^3 Xo(3)

X_e(0) = Xee(0) + W_8^0 Xeo(0) ; X_e(2) = Xee(0) - W_8^0 Xeo(0)

Xe(1) = Xee(1) + W_8^2 Xeo(1) ; Xe(3) = Xee(1) - W_8^2 Xeo(1)

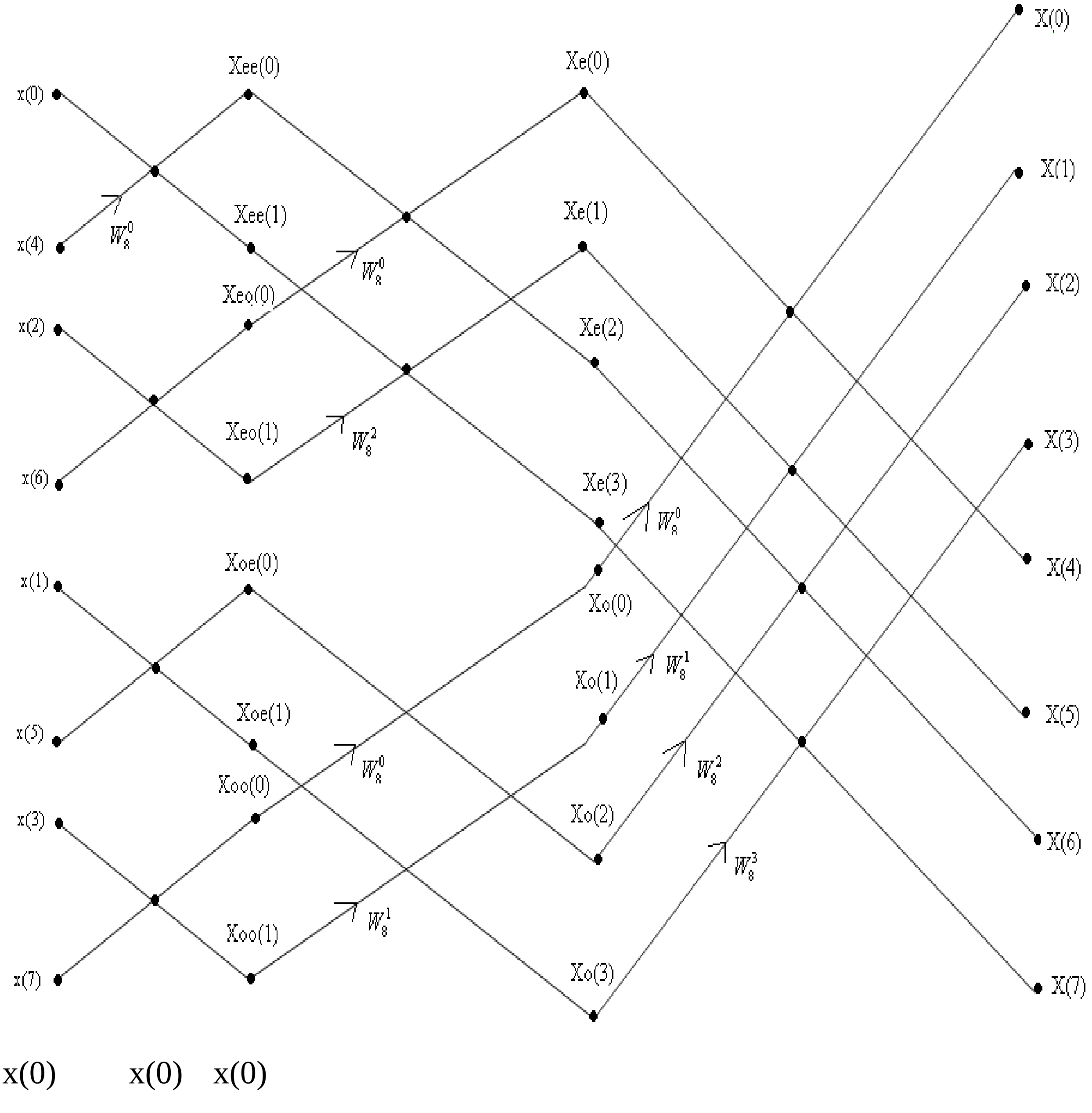
Xo(0) = Xoe(0) + W_8^0 Xoo(0) ; Xo(2) = Xoe(0) - W_8^0 Xoo(0)

Xo(1) = Xoe(1) + W_8^2 Xoo(1) ; Xo(3) = Xoe(1) - W_8^2 Xoo(1)

Xee(k) = $\sum_{0}^{\frac{N}{4}-1} x(4n)W_N^{4nk} = \sum_{n=0}^{1} x(4n)W_N^{4nk}$ =x(0) + x(4) W_8^{4k}

Xee(0) = x(0)+x(4)

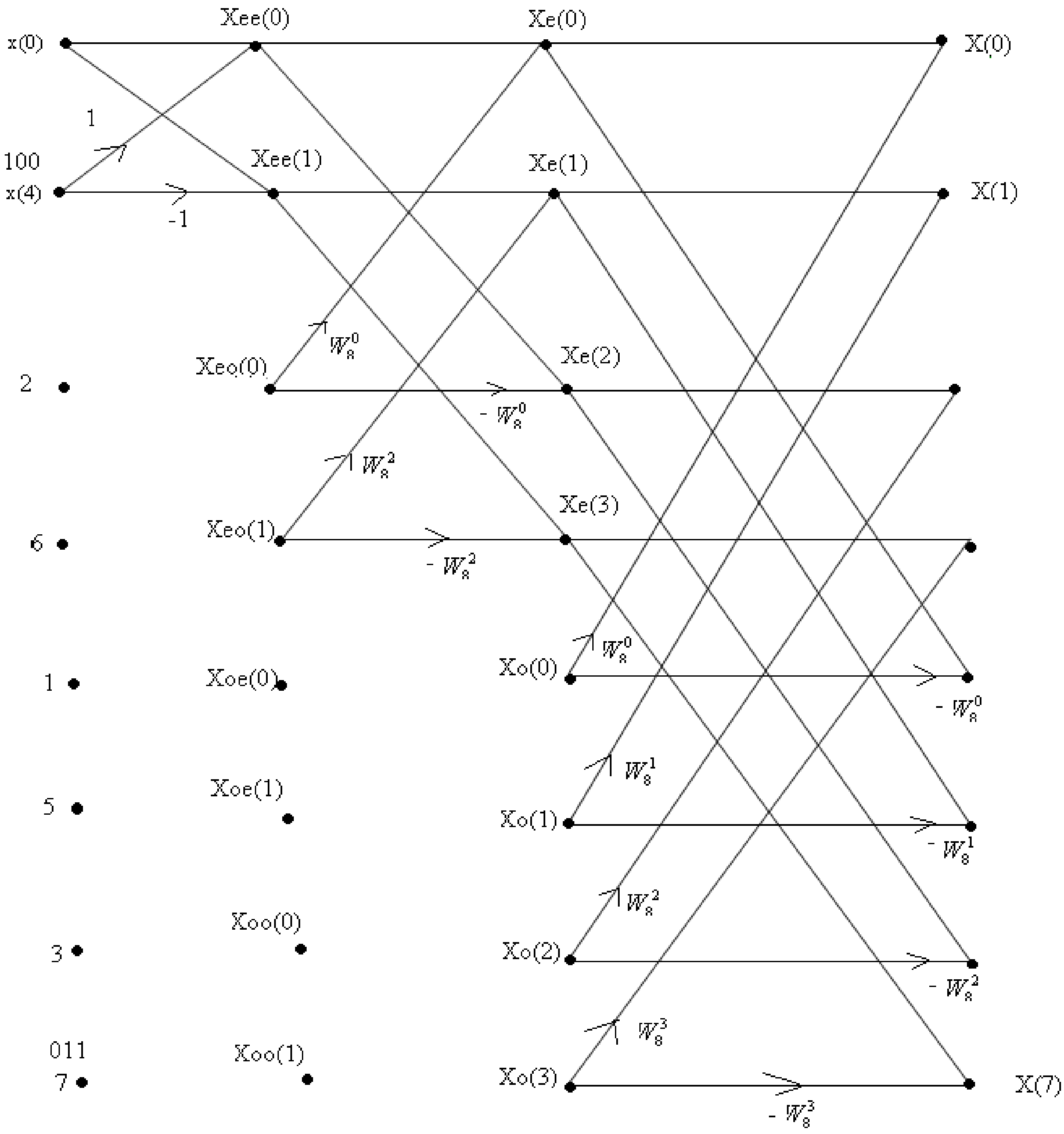
Xee(1) = x(0)-x(4)



x(0)	x(0)	x(0)
<u>x(4)</u>	x(2)	x(1)
x(2)	x(4)	x(2)
<u>x(6)</u>	<u>x(6)</u>	x(3)
x(1)	x(1)	x(4)

x(5)	x(3)	x(5)
x(3)	x(5)	x(6)
x(7)	x(7)	x(7)

Other way of representation



5.2 **DIFFFT:**

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)W_N^{nk} + \sum_{n^1=N/2}^{N-1} x(n')W_N^{n'k} \qquad \text{put } n' = n+N/2$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n)W_N^{nk} + \sum_{n=0}^{\frac{N}{2}-1} x(n+N/2)W_N^{(n+N/2)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(n)W_N^{nk} + W_N^{\frac{N}{2}k} \sum_{n=0}^{\frac{N}{2}-1} x(n+N/2)W_N^{nk}$$

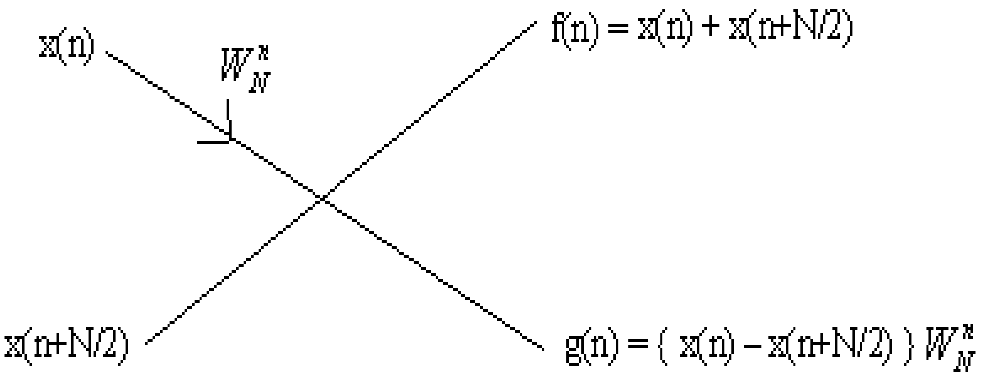
$$= \sum_{n=0}^{\frac{N}{2}-1} [x(n) + (-1)^k\, x(n+\frac{N}{2})]\, W_N^{nk}$$

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} [x(n) + x(n+\frac{N}{2})]\, W_{N/2}^{nk}$$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} \{[x(n) - x(n+\frac{N}{2})]\, W_N^{n}\}\, W_{N/2}^{nk}$$

Let f(n) = x(n) + x(n+N/2)

g(n) = { x(n) – x(n+N/2) } W_N^n



N=8

f(0) = x(0) + x(4)

f(1) = x(1) + x(5)

f(2) = x(2) + x(6)

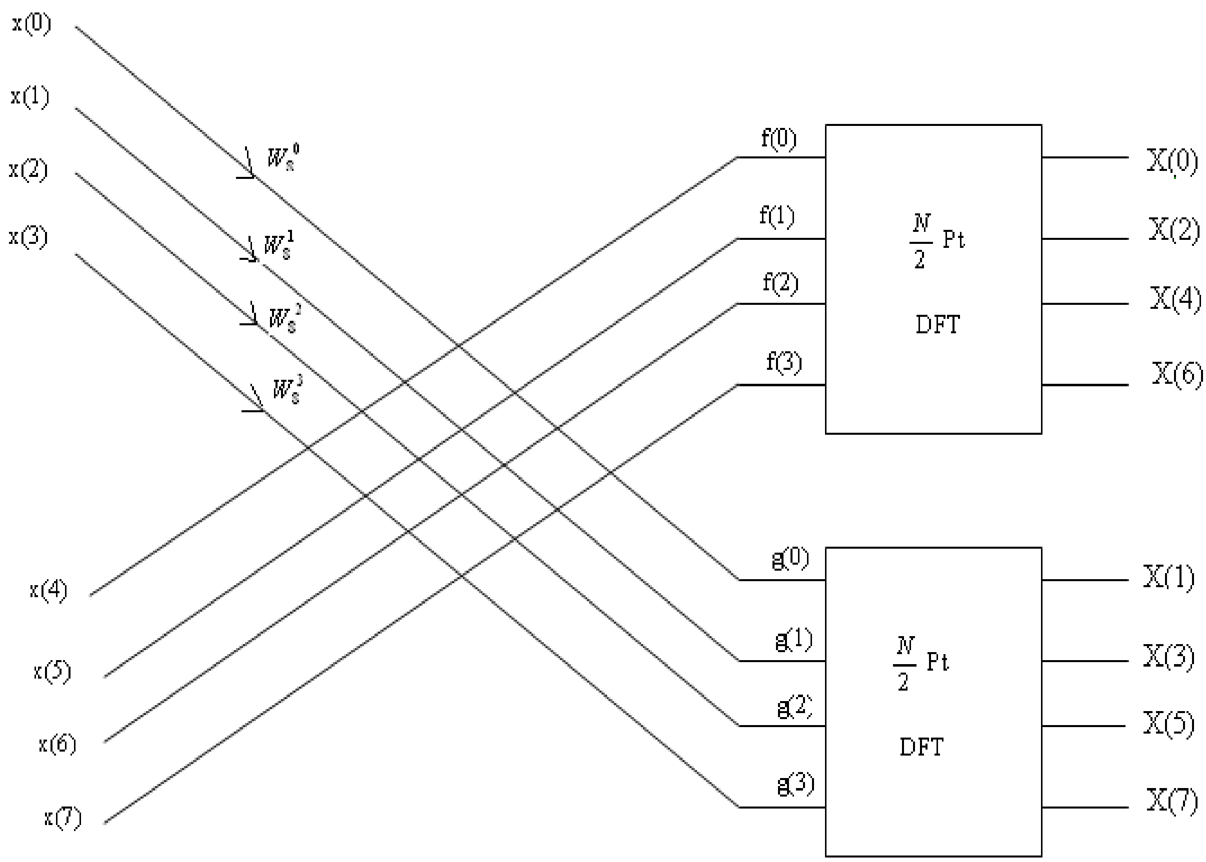
f(3) = x(3) + x(7)

g(0) = [x(0) - x(4)] W_8^0

g(1) = [x(1) - x(5)] W_8^1

g(2) = [x(2) - x(6)] W_8^2

g(3) = [x(3) - x(7)] W_8^3



$$X(4k) = \sum_{n=0}^{\frac{N}{4}-1} [f(n) + f(n+\tfrac{N}{4})] W_{N/4}^{nk}$$

$$X(4k+2) = \sum_{n=0}^{\frac{N}{4}-1} [\{f(n) - f(n+\tfrac{N}{4})\} W_{N/2}^{n}] W_{N/4}^{nk}$$

$$X(4k+1) = \sum_{n=0}^{\frac{N}{4}-1} [g(n) + g(n+\tfrac{N}{4})] W_{N/4}^{nk}$$

$$X(4k+3) = \sum_{n=0}^{\frac{N}{4}-1} [\{g(n) - g(n+\tfrac{N}{4})\} W_{N/2}^{nk}] W_{N/4}^{nk}$$

X(4k) = f(0) + f(2) + [f(1) + f(3)] W_8^{4k}

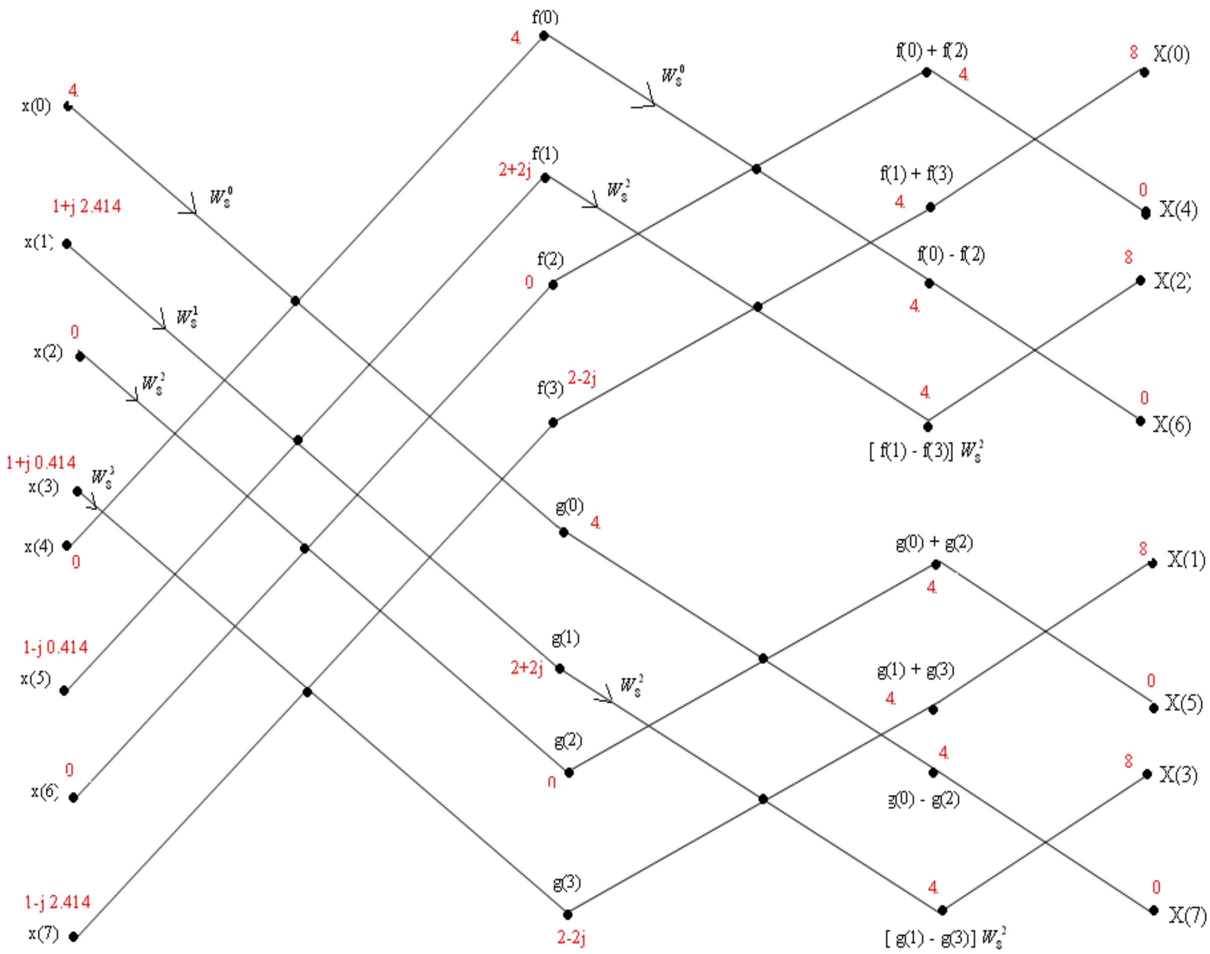
X(4k+2) = f(0) – f(2) + { [f(1) – f(3)] W_8^{2} } W_8^{4k}

X(0) = f(0) + f(2) + f(1) + f(3)

X(4) = f(0) + f(2) – [f(1) + f(3)]

X(2) = f(0) - f(2) + [f(1) - f(3)] W_8^2

X(6) = f(0) - f(2) - [f(1) - f(3)] W_8^2



Find the IDFT using DIFFFT

X(k) = { 4, 1-j 2.414, 0, 1-j 0.414, 0, 1+j 0.414, 0, 1+j 2.414 }

Out put 8x*(n) is in bit reversal order x(n) = { 1,1,1,1,0,0,0,0}

6.DIGITAL FILTER STRUCTURE

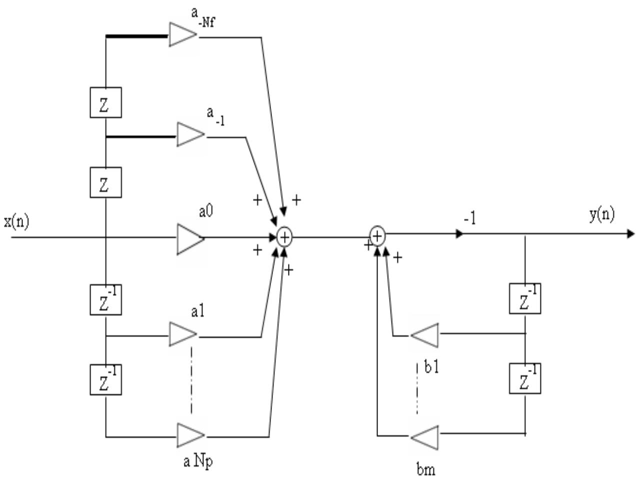
The difference equation

$$y(n) = \sum_{k=-N_F}^{N_P} a_k\, x(n-k) + \sum_{k=1}^{M} b_k\, y(n-k)$$

$$H(z) = \frac{\sum_{k=-N_F}^{N_P} a_k z^{-k}}{1-\sum_{k=1}^{M} b_k z^{-k}} \quad \text{or} \quad = A\, Z^{N_F} \prod_{k=1}^{Np+NF} \frac{(1-C_k)Z^{-1}}{\prod_{k=1}^{M}(1-d_k)Z^{-1}}$$

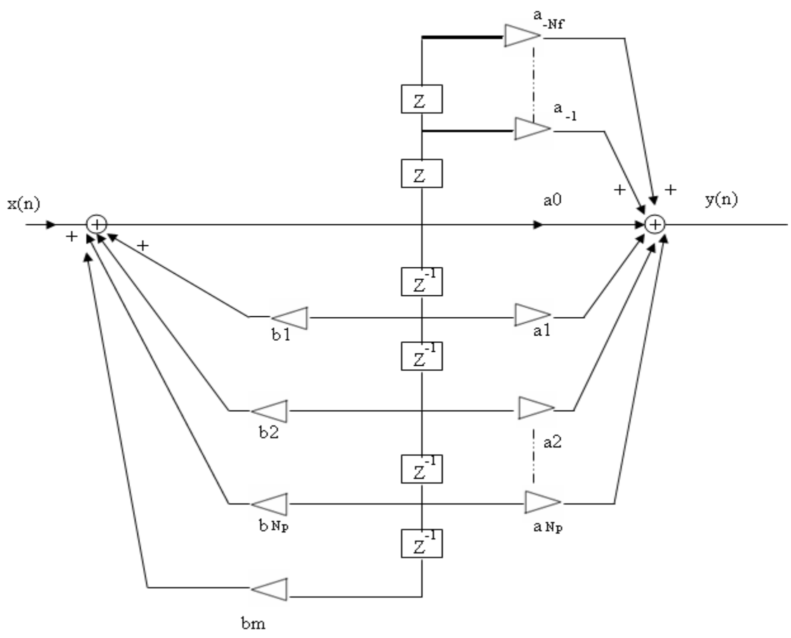
If b_k= 0 non recursive or all zero filter.

6.1 Direct Form – I



1. Easily implemented using computer program.
2. Does not make most efficient use of memory = M+Np+N_F delay elements.

6.2 Direct form-II



Smaller no. of delay elements = Max of (M, Np) + N_F

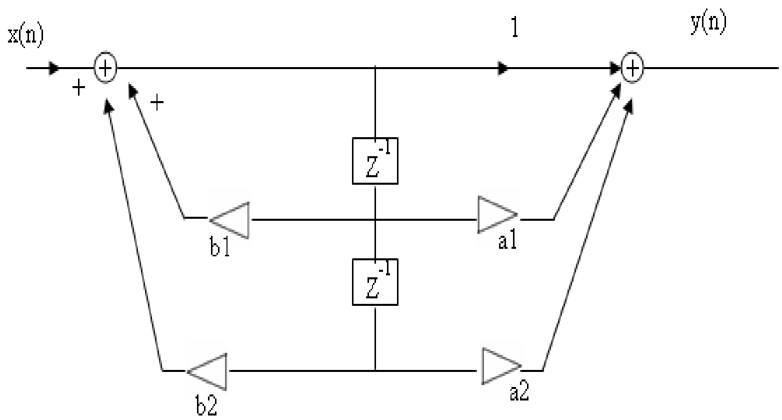
Disadvantages of D-I & D-II

1. They lack hardware flexibility, in that, filters of different orders, having different no. of multipliers and delay elements.
2. Sensitivity of co-efficient to quantization effects that occur when using finite-precision arithmetic.

6.3 Cascade Combination of second-order section (CSOS)

$y(n) = x(n) + a_1\, x(n-1) + a_2\, x(n-2) + b_1\, y(n-1) + b_2\, y(n-2)$

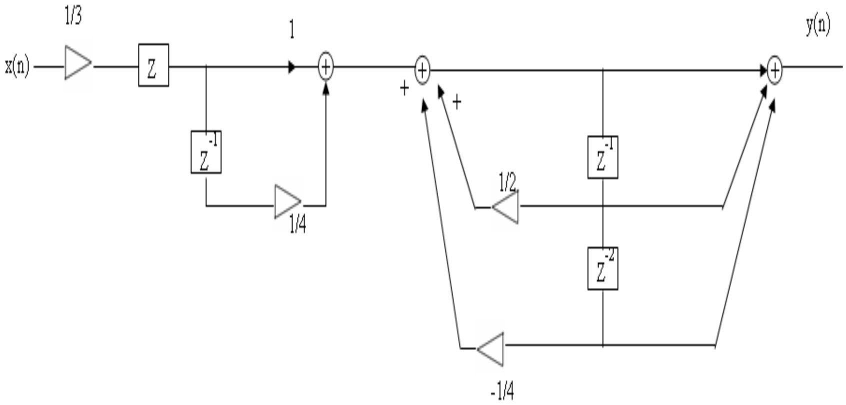
$$H(z) = \frac{1 + a_1 Z^{-1} + a_2 Z^{-2}}{1 - b_1 Z^{-1} - b_2 Z^{-2}}$$



Ex:

$$H(z) = \frac{\frac{z}{3}+\frac{5}{12}+\frac{5}{12}Z^{-1}+\frac{Z^{-2}}{12}}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}} = \frac{\frac{z}{3}\left[1+\frac{5}{4}Z^{-1}+\frac{5Z^{-2}}{4}+\frac{1}{4}Z^{-3}\right]}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}}$$

$$= \frac{\frac{z}{3}\left[1+\frac{1}{4}Z^{-1}\right]\left[1+Z^{-1}+Z^{-2}\right]}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}} = \frac{1}{3}\ z\left[1+\frac{1}{4}Z^{-1}\right]\frac{\left[1+Z^{-1}+Z^{-2}\right]}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}}$$

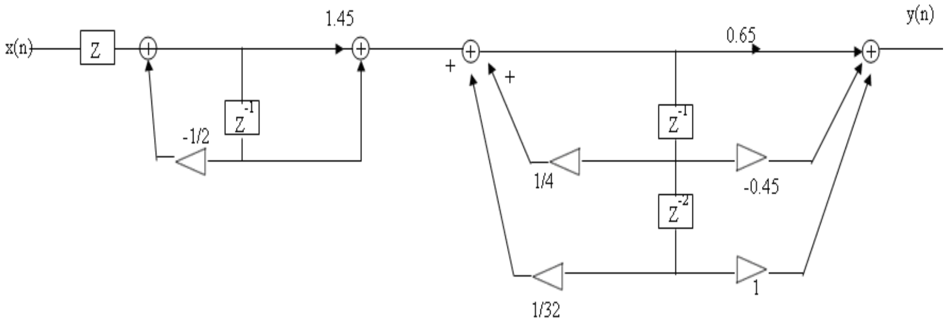


Ex:

$$H(z) = \frac{\left[Z+Z^{-1}+Z^{-2}\right]}{\left[1+\frac{Z^{-1}}{2}\right]\left[1-\frac{Z^{-1}}{4}\right]\left[1+\frac{Z^{-1}}{8}\right]} = Z\frac{\left[1+Z^{-2}+Z^{-3}\right]}{\left[1+\frac{Z^{-1}}{2}\right]\left[1-\frac{Z^{-1}}{4}\right]\left[1+\frac{Z^{-1}}{8}\right]}$$

$$= Z\frac{\left[0.65-0.45Z^{-1}+Z^{-2}\right]\left[1.45+Z^{-1}\right]}{\left[1+\frac{Z^{-1}}{2}\right]\left[1-\frac{Z^{-1}}{4}\right]\left[1+\frac{Z^{-1}}{8}\right]}$$

$$= Z\ \frac{\left[1.45+Z^{-1}\right]}{\left[1+\frac{Z^{-1}}{2}\right]}\ \frac{\left[0.65-0.45Z^{-1}+Z^{-2}\right]}{1-\frac{Z^{-1}}{4}-\frac{Z^{-2}}{32}}$$



6.4 Parallel Combination of Second Order Section (PSOS)

Ex:

$$H(z) = \frac{\frac{z}{3}+\frac{5}{12}+\frac{5}{12}Z^{-1}+\frac{Z^{-2}}{12}}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}} \quad = \quad \frac{Z\left[\frac{1}{3}+\frac{5}{12}Z^{-1}+\frac{5}{12}Z^{-2}+\frac{Z^{-3}}{12}\right]}{1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}}$$

$$1-\frac{Z^{-1}}{2}+\frac{Z^{-2}}{4}\ \Big]\frac{Z^{-3}}{12}+\frac{5}{12}Z^{-2}+\frac{1}{3}+\frac{5}{12}Z^{-1}++\Big[\ \frac{Z^{-1}}{3}+\frac{7}{3}$$

$$\frac{Z^{-3}}{12}-\frac{Z^{-2}}{6}+\frac{Z^{-1}}{3}$$

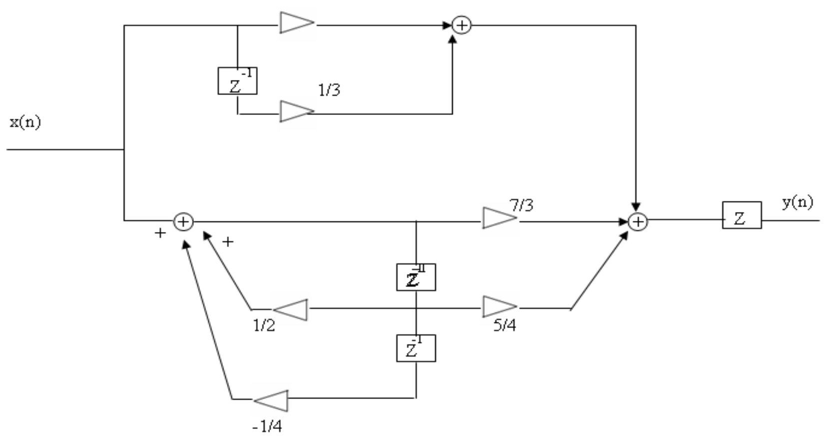
___-____+____-_______

$$\frac{7Z^{-2}}{12}+\frac{Z^{-1}}{12}+\frac{1}{3}$$

$$\frac{7Z^{-2}}{12}-\frac{7Z^{-1}}{6}+\frac{7}{3}$$

$$\frac{5}{4}Z^{-1}+\frac{7}{3}$$

$$H(z) = Z\left[-2+\frac{Z^{-1}}{3}+\frac{\frac{7}{3}+\frac{5}{4}Z^{-1}}{1-Z^{-1}/2+\frac{Z^{-2}}{4}}\right]$$

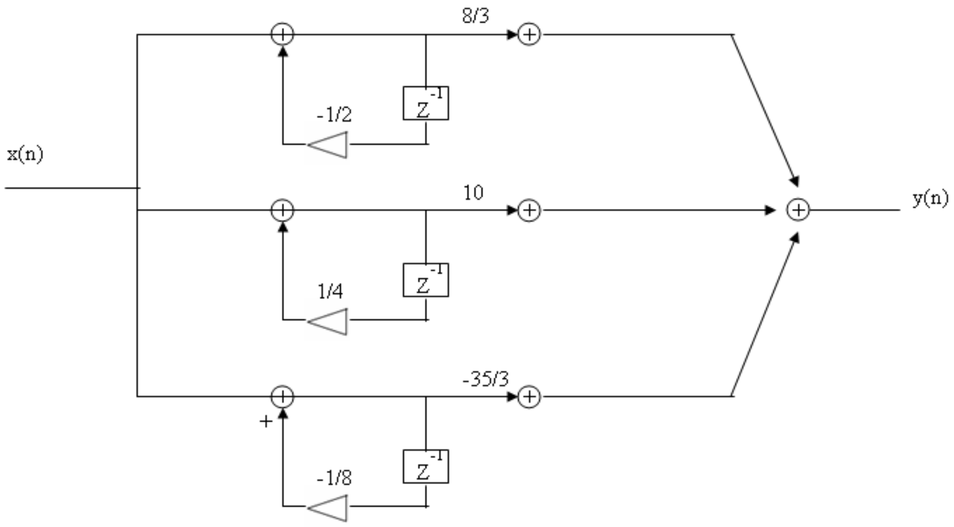


Ex:

$$H(z) = \frac{\left[Z + Z^{-1} + Z^{-2}\right]}{\left[1+\frac{Z^{-1}}{2}\right]\left[1-\frac{Z^{-1}}{4}\right]\left[1+\frac{Z^{-1}}{8}\right]} \quad \text{obtain PSOS}$$

$$\frac{\left[1+Z^{-1}\right]\left[1+2Z^{-1}\right]}{\left[1+\frac{Z^{-1}}{2}\right]\left[1-\frac{Z^{-1}}{4}\right]\left[1+\frac{Z^{-1}}{8}\right]} = \frac{A}{\left[1+\frac{Z^{-1}}{2}\right]} + \frac{B}{\left[1-\frac{Z^{-1}}{4}\right]} + \frac{C}{\left[1+\frac{Z^{-1}}{8}\right]}$$

A = 8/3 B = 10 C = -35/3



6.5 Jury – Stability Criterion

$$H(z) = \frac{N(z)}{D(z)}$$

$$D(z) = \sum_{i=0}^{N} b_i Z^{N-i} = b_o Z^N + b_1 Z^{N-1} + b_2 Z^{N-2} + \ldots b_{N-1} Z^1 + b_N$$

ROWS	COEFFICIENTS
1	b_o b_1 ……. b_N
2	b_N b_{N-1} ……. bo
3	C_o C_1 ……. C_{N-1}
4	C_{N-1} C_{N-2} ……. Co
5	d_o d_1 ……. d_{N-2}
6	d_{N-2} d_{N-3} ……. do

. . . 2N-3	 r_0 r_1 r_2

$C_i = \begin{vmatrix} b_o & b_{N-i} \\ b_N & b_i \end{vmatrix}$ i = 0,1,…N-1

$d_i = \begin{vmatrix} c_o & c_{N-1-i} \\ c_{N-1} & c_i \end{vmatrix}$ i = 0,1,…N-2

i. D(1) > 0
ii. $(-1)^N$ D(-1) > 0
iii. $|b_o| > |b_N|$ $|c_o| > |c_{N-1}|$ $|d_o| > |d_{N-2}|$ $|r_o| > |r_2|$

Ex:

H(z) = $\dfrac{Z^4}{4Z^4 + 3Z^3 + 2Z^2 + Z + 1}$ D(z) = $4Z^4 + 3Z^3 + 2Z^2 + Z + 1$

1 2	4 3 2 1 1 1 1 2 3 4
3 4	15 11 6 1 1 6 11 15
5	224 159 79

D(1) = 4+3+2+1+1 = 11 > 0, $(-1)^4$ D(-1) = 3 >0

$|b_o| > |b_4|$ $|c_o| > |c_3|$ $|d_o| > |d_2|$ Stable.

Ex:

H(z) = $\dfrac{1}{1 - \frac{7}{4}Z^{-1} - \frac{1}{2}Z^{-2}} = \dfrac{4Z^2}{4Z^2 - 7Z - 2}$ Ans: Unstable

Non Recursive filters	Recursive filters
$y(n) = \sum_{k=-\infty}^{\infty} a_k\, x(n-k)$ for causal system $= \sum_{k=0}^{\infty} a_k\, x(n-k)$ For causal i/p sequence $y(n) = \sum_{k=0}^{N} a_k\, x(n-k)$ It gives FIR o/p. All zero filter. Always stable.	$y(n) = \sum_{k-Nf}^{Np} a_k\, x(n-k) - \sum_{k=1}^{M} b_k\, y(n-k)$ for causal system $y(n) = \sum_{k0}^{Np} a_k\, x(n-k) - \sum_{k=1}^{M} b_k\, y(n-k)$ It gives IIR o/p but not always. Ex: y(n) = x(n) – x(n-3) + y(n-1) General TF : $H(z) = \dfrac{\sum_{k=-N_F}^{N_P} a_k z^{-k}}{1-\sum_{k=1}^{M} b_k z^{-k}}$ bk = 0 for Non Recursive Nf= 0 for causal system

FIR filters	IIR filters
1. Linear phase no phase distortion.	Linear phase, phase distortion.
2. Used in speech processing, data transmission & correlation processing	Graphic equalizers for digital audio, tone generators filters for digital telephone
3. Realized non recursively.	Realized recursively.
4. stable	Stable or unstable. $H(n) = a^n u(n)$ a<1 stable = 0 a>1 unstable
5. filter order is more	Less
6. more co-efficient storage	Less storage
7. Quantization noise due to finite precision arithmetic can be made negligible	Quantization noise
8. Co-efficient accuracy problem is	More

less severe	
9. used in multirate DSP (variable sampling rate)	

7. IIR FILTER DESIGN

- Butterworth, chebyshev & elliptic techniques.
- Impulse invariance and bilinear transformation methods are used for translating s-plane singularities of analog filter to z-plane.
- Frequency transformations are employed to convert LP digital filter design into HP, BP and BR digital filters.
- All pass filters are employed to alter only the phase response of IIR digital filter to approximate a linear phase response over the pass band.

The system function = H(s)

The frequency transfer function = H(jΩ) = H(s) / s=jΩ

The power transfer function = $|H(j\Omega)|^2$ = H(jΩ) H*(jΩ) = H(s) H(-s) / s=jΩ

To obtain the stable system, the polse that lie in the left half of the s-plane are assigned to H(s).

7.1 BUTTERWORTH FILTER DESIGN

The butterworth LP filter of order N is defined as H_B**(s)** H_B**(-s) =** $\dfrac{1}{1+\left(\dfrac{s}{j\Omega_c}\right)^{2N}}$

Where s = jΩ_c

$|H_B(j\Omega_c)|^2 = \frac{1}{2}$ or $|H_B(j\Omega_c)|\ db$ = -3dB 's

It has 2N poles

$$1+\left(\frac{s}{j\Omega_c}\right)^{2N} = 0$$

$$\left(\frac{s}{j\Omega_c}\right)^{2N} = -1$$

$S^{2N} = -1\ (j\Omega_c)^{2N}$

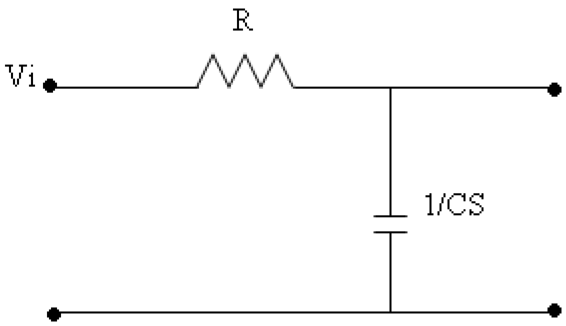
$$= e^{j\pi}(e^{j\frac{\pi}{2}}\Omega_c)^{2N} = \Omega_c^{\ 2N}\ e^{j\pi}\ e^{j\frac{\pi}{2}2N}\ e^{j2m\pi}$$

$$S^{2N} = \Omega_c^{\ \overline{2N}}\ e^{j\pi\left(\frac{1+N+2m}{}\right)}$$

$S_m = \Omega_c \; e^{j\pi\left(\frac{1+N+2m}{2N}\right)} \qquad 0 \le m \le 2N-1$

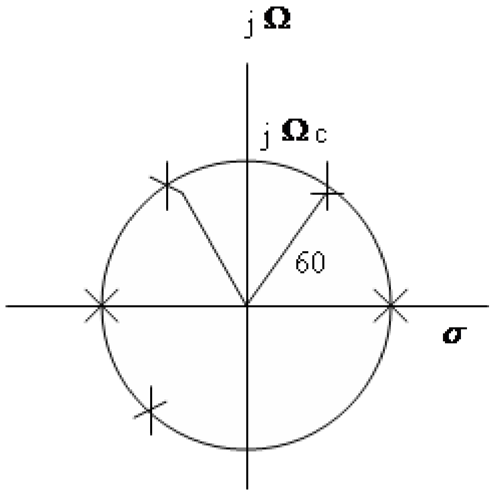
Ex: for N=3

$e^{j\pi\frac{(4+2m)}{6}} = e^{j\frac{2\pi}{3}}, e^{j\pi}, e^{j\frac{4\pi}{3}}, e^{j\frac{5\pi}{3}}, e^{j2\pi}, e^{j\frac{7\pi}{3}}$ = $120^0, 180^0, 240^0, 300^0, 360^0, 60^0$



$$\frac{Vo(s)}{Vi(s)} = \frac{\frac{1}{CS}}{R+\frac{1}{CS}} = \frac{1}{1+RCS}$$

$$\Omega_c = \frac{1}{RS} = \frac{1}{1+\frac{s}{\Omega_c}} = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^2}}$$



Poles that are let half plane are belongs to desired system function.

$$|H_B(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}$$

For a large Ω, magnitude response decreases as Ω^{-N}, indicating the LP nature of this filter.

$$|H_B(j\Omega)|_{dB} = 10\log_{10}|H_B(j\Omega)|^2$$

$$= -10\log_{10}\left(1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}\right)$$

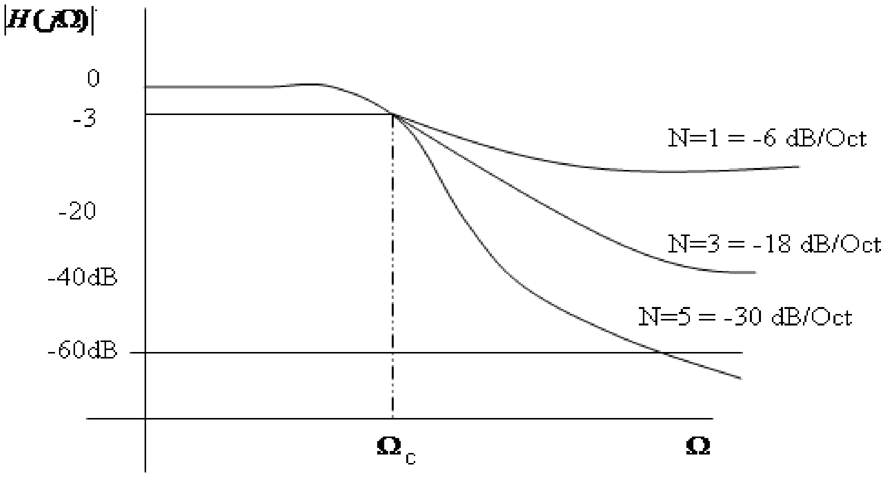
As $\Omega \rightarrow \infty$

$= -20 \text{ N } \log_{10}\Omega$

$= -20$ N dB/ Decade $= -6$ N dB/Octane

As N increases, the magnitude response approaches that of ideal LP filter.

The value of N is determined by Pass & stop band specifications.



Ex: Design Butterworth LPF for the following specifications.

Pass band:

$-1 < |H(j\Omega)|^2$ dB ≤ 0 for $0 \leq \Omega \leq 1404\pi$ $(\Omega_p = 1404\pi)$

Stop band:

$|H(j\Omega)|^2$ dB < -60 for $\Omega \geq 8268\pi$ $(\Omega_s = 8268\pi)$

If the Ω_c is given

$$|H(j\Omega s)|^2 = [1+\left(\frac{\Omega s}{\Omega_c}\right)^{2N}]^{-1} < 10^{-6} \text{ (-60dB)}$$

$$= N > \frac{\log(10^6 - 1)}{2\log(\frac{\Omega s}{\Omega c})}$$

Since Ω_c is not given, a guess must be made.

The specifications call for a drop of -59dB, In the frequency range from the edge of the pass band (1404π) to the edge of stop band (8268π). The frequency difference is equal to $\log_2\left(\frac{8268}{1404}\right) = 2.56$ octaves.

1 oct ---- - 6N dB

2.56 ------ ?

=> 2.56 X - 6N dB = -59 dB's

N = $\frac{-59}{-2.56X6} = 3.8$

There fore: N =4

Now $\left|H_B(j\Omega s)\right|^2$ = [$1+\left(\frac{\Omega s}{\Omega_c}\right)^{2N}$]$^{-1}$ < 10^{-6}

$1+\left(\frac{\Omega s}{\Omega_c}\right)^{2N}$ > 10^6

Ωs^{2N} > 10^6 Ωc^{2N}

Ωs $10^{\frac{-6}{2N}}$ > Ωc => 1470.3π > Ωc

Ωc <1470.3π

Let Ωc =1470.3π

At this Ωc it should satisfy pass band specifications.

$\left|H_B(j\Omega p)\right|^2$ = [$1+\left(\frac{\Omega p}{\Omega_c}\right)^{2N}$]$^{-1}$ > 0.794 (= -1dB)

= 0.59

This result is below the pass band specifications. Hence N=4 is not sufficient.

Let N=5

Ωc < Ωs X $10^{\frac{-6}{2N}}$ = 2076.8π

In the pass band $\left|H_B(j\Omega p)\right|^2$ = [$1+\left(\frac{1404}{2076}\right)^{10}$]$^{-1}$ = 0.98

Since N=5

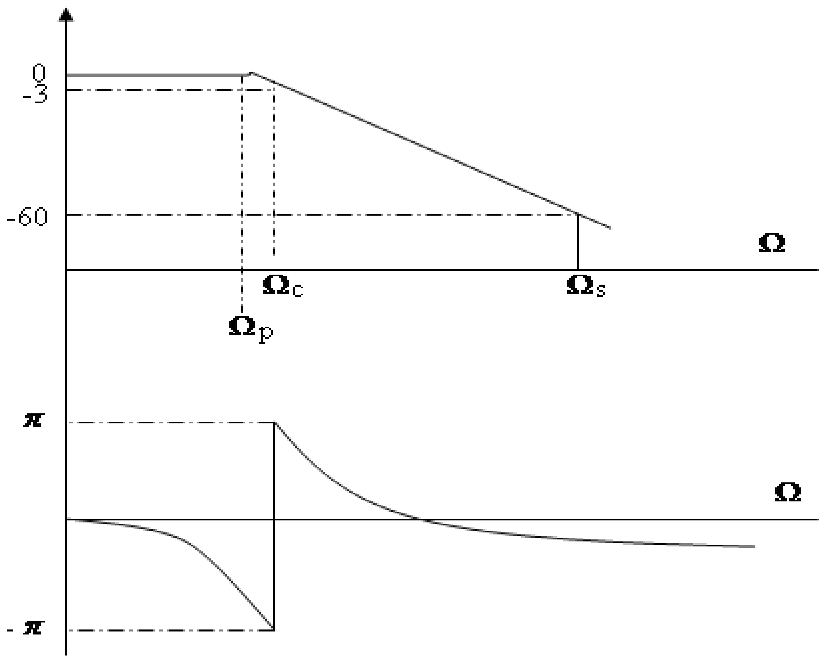
Ωc = 2076π

S_1 = -2076π

$S_{2,3}$ = 2076π (cos (4π/5) $\pm$j sin(4π/5)) = 2076π $e^{\pm j144}$

$S_{4,5}$ = 2076π (cos (3π/5) $\pm$j sin(3π/5)) = 2076π $e^{\pm j108}$

$$H_B(s) = \frac{(2076\pi)^5}{[s+2076\pi][s^2+3359\pi s+(2076\pi)^2][s^2+1283\pi s+(2076\pi)^2]}$$



1. Magnitude response is smooth, and decreases monotonically as Ω increases from 0 to ∞

2. the magnitude response is maximally flat about $\Omega=0$, in that all its derivatives up to order N are equal to zero at $\Omega=0$

Ex: $\Omega c=1$, N=1

$|H_B(j\Omega)|^2 = (1+\Omega^2)^{-1}$

The first derivative

$$\frac{d}{d\Omega}|H_B(j\Omega)|^2 = \frac{-2\Omega}{(1+\Omega^2)^2} = 0 \text{ at } \Omega=0$$

The second derivative

$$\frac{d^2}{d\Omega^2}|H_B(j\Omega)|^2 = -2 \text{ at } \Omega=0$$

3. The phase response curve approaches $\frac{-N\pi}{2}$ for large Ω, where N is the no. of poles of butterworth circle in the left side of s-plane.

Advantages:

1. easiest to design
2. used because of smoothness of magnitude response .

Disadvantage:

Relatively large transition range between the pass band and stop band.

Other procedure

When $\Omega c = 1$ Avs = $\dfrac{Avo}{1+\left(\dfrac{w}{wo}\right)^{2N}}$

$$|H_B(s)|^2 = \frac{Avo}{1+\left(\dfrac{s}{j}\right)^{2N}}$$

If n is even $S^{2N} = 1 = e^{j(2k-1)\pi}$

The 2N roots will be Sk= $e^{j(2k-1)\frac{\pi}{2N}}$ k=1,2,….2N

Sk $= Cos(2k-1)\dfrac{\pi}{2N} + jSin(2k-1)\dfrac{\pi}{2N}$

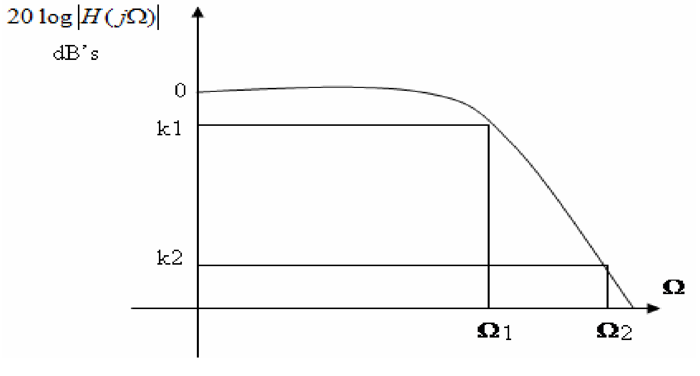
Therefore: $|H_B(s)|^2$ = T(s) = $\dfrac{1}{\prod_{k=1}^{N/2}(s^2+2Cos\theta_k s+1)}$ where $\theta_k = (2k-1)\dfrac{\pi}{2N}$

If N is odd

$S^{2n} = 1 = e^{j2k\pi}$

Sk = $e^{j2k\pi/N}$ k=0,1,2….(2N-1)

T(s) = $\dfrac{1}{\prod_{k=1}^{(N-1)/2}(s^2+2Cos\theta_k s+1)}$ where $\theta_k = k\dfrac{\pi}{N}$



$0 \geq 20\log|H(j\Omega)| \geq K1$ for $\Omega \leq \Omega 1$

$20\log|H(j\Omega)| \leq K2$ for $\Omega \geq \Omega 2$

$$10\log\left[\frac{1}{1+\left(\frac{\Omega 1}{\Omega c}\right)^{2N}}\right] = K1 \qquad \left(\frac{\Omega 1}{\Omega c}\right)^{2n} = 10^{\frac{-k1}{10}} - 1$$

$$10\log\left[\frac{1}{1+\left(\frac{\Omega 2}{\Omega c}\right)^{2N}}\right] = K2 \qquad \left(\frac{\Omega 2}{\Omega c}\right)^{2n} = 10^{\frac{-k2}{10}} - 1$$

Dividing $\left(\frac{\Omega 1}{\Omega 2}\right)^{2n} = \dfrac{10^{\frac{-k1}{10}} - 1}{10^{\frac{-k2}{10}} - 1}$

$$n = \frac{\log_{10}\left[\dfrac{10^{\frac{-k1}{10}} - 1}{10^{\frac{-k2}{10}} - 1}\right]}{2\log_{10}\left(\dfrac{\Omega 1}{\Omega 2}\right)}$$

choosing this value for n, results in two different selections for Ω_c. If we wish to satisfy our requirement at $\Omega 1$ exactly and do better than our req. at $\Omega 2$, we use

$\Omega_c = \dfrac{\Omega_1}{\left(10^{-\frac{k1}{10}} - 1\right)^{\frac{1}{2n}}}$ or $\Omega_c = \dfrac{\Omega_2}{\left(10^{-\frac{k2}{10}} - 1\right)^{\frac{1}{2n}}}$ for better req at $\Omega 2$

End

7.2 CHEBYSHEV FILTER DESIGN

Defined as $H_c(S)\, H_c(-S) = \left[1 + \mu^2 C_N^2\left(\frac{S}{j\Omega p}\right)\right]^{-1}$

μ = measure of allowable deviation in the pass band.

$C_N(x) = \mathrm{Cos}(N\mathrm{Cos}^{-1}(x))$ is the Nth order polynomial.

Let x = Cosθ

$C_N(x) = \mathrm{Cos}(N\theta)$

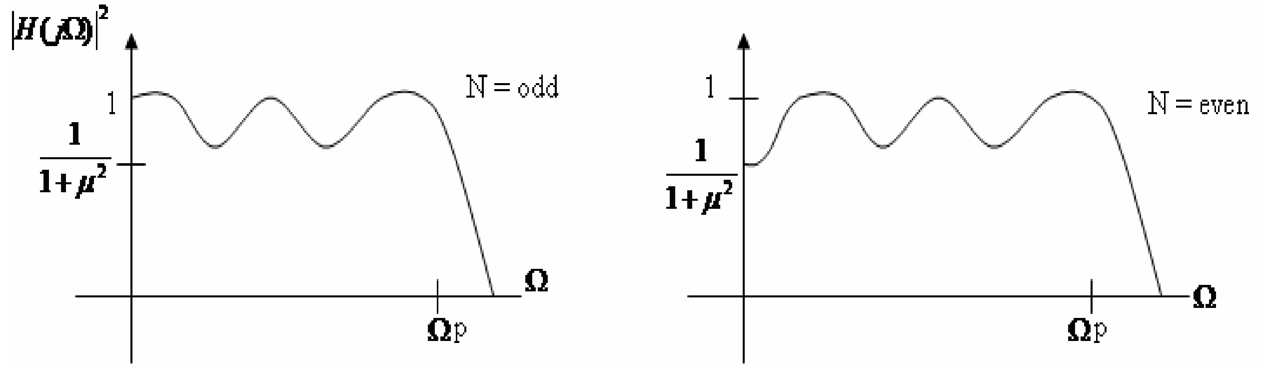
$C_0(x) = 1$

$C_1(x) = \mathrm{Cos}\theta = x$

$C_2(x) = \mathrm{Cos}2\theta = 2\,\mathrm{Cos}^2\theta - 1 = 2x^2 - 1$

$C_3(x) = \mathrm{Cos}3\theta = 4\,\mathrm{Cos}^3\theta - 3\,\mathrm{Cos}\theta = 4x^3 - 3x$ etc..

N	$C_N(x)$
0	1
1	x
2	$2x^2-1$
3	$4x^3-3x$
4	$8x^4- 8x^2 +1$



Two features of Chebyshev poly are important for the filter design

1. $|C_N(x)| \leq 1$ for $|x| \leq 1$

$$(1+\mu^2)^{-1} \leq |H_c(j\Omega)|^2 \leq 1 \qquad \text{for} \quad 0 \leq \Omega \leq \Omega p$$

Transfer function lies in the range $(1+\mu^2)^{-1} \leq |H_c(j\Omega)|^2 \leq 1$ for $0 \leq \Omega \leq \Omega p$

Whereas the frequency value important for the design of the Butterworth filter was the Ωc, the relevant frequency for the Chebyshev filter is the edge of pass band Ωp.

2. $|x| >> 1, |C_N(n)|$ Increases as the Nth power of x. this indicates that for $\Omega >> \Omega p$, the magnitude response decreases as Ω^{-N}, or -6N dB Octane. This is identical to Butterworth filter.

Now the ellipse is defined by major & minor axis.

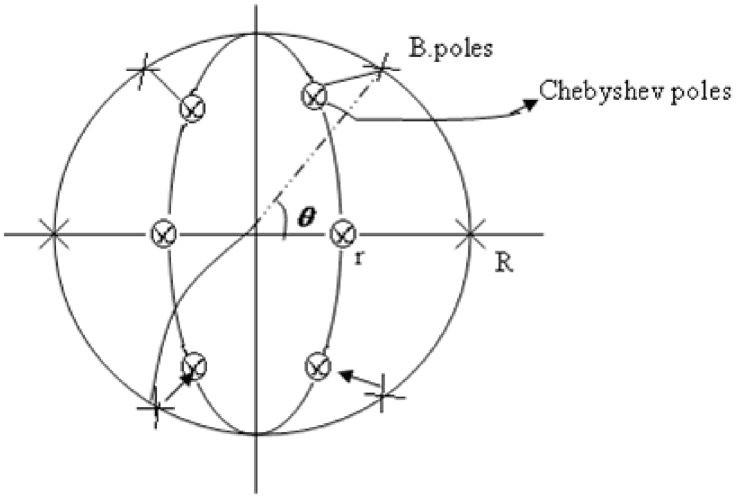
Define $\rho = \mu^{-1} + \sqrt{1+\mu^{-2}}$

$$\text{Minor } r = \Omega p \frac{\left(\rho^{\frac{1}{N}} - \rho^{\frac{-1}{N}}\right)}{2}$$

$$\text{Major } R = \Omega p \frac{\left(\rho^{\frac{1}{N}} + \rho^{\frac{-1}{N}}\right)}{2}$$

N = Order of filter.

$S_P = r \cos\theta + j R \sin\theta$



Ex:

Pass band:

-1< $|H(j\Omega)|^2$ dB $\leq$ 0 for $0 \leq \Omega \leq 1404\pi$

Stop band:

$|H(j\Omega)|^2$ dB < -60 for $\Omega \geq 8268\pi$

Value of μ is determined from the pass band

10 log$\left(1+\mu^2\right)^{-1}$ > -1dB -1dB = 0.794

$\mu < \left[10^{0.1}-1\right]^{\frac{1}{2}}$ = 0.508

μ = 0.508

Value of N is determined from stop band inequality

$$\left|H_c\left(j\Omega s\right)\right|^2 = \left[1+\mu^2 C_N^2\left(\frac{s}{j\Omega p}\right)^{-1}\right] < 10^{-6}$$

Since $\frac{\Omega s}{\Omega p} = 5.9$ $C_N(5.9) > \left[\frac{\left(10^6-1\right)}{\mu^2}\right]^{\frac{1}{2}}$ = 1969

Evaluating

$C_3(5.9)$ = 804 $C_4(5.9)$ = 9416 therefore N = 4 is sufficient.

Since this last inequality is easily satisfied with N=4 the value of μ can be reduced to as small as 0.11, to decrease pass band ripple while satisfying the stop band. The value μ=0.4 provides a margin in both the pass band and stop band. We proceed with the design with μ =0.508 to show the 1dB ripple in the pass band.

Axes of Ellipse:

ρ =0.508^{-1} + (1+0.508^{-2})$^{1/2}$ = 4.17

R = $\frac{1404\pi}{2}\left[4.17^{\frac{1}{4}} + 4.17^{\frac{-1}{4}}\right] = 702\pi(1.43 + 0.67) = 1494\pi$

r = $\frac{1404\pi}{2}\left[4.17^{\frac{1}{4}} - 4.17^{\frac{-1}{4}}\right] = 512\pi$

poles locations : $\pm\frac{7\pi}{8}, \pm\frac{5\pi}{8}$

$S_{1,2} = 512\pi Cos\frac{7\pi}{8} \pm j1494\pi Sin\frac{7\pi}{8} = -473\pi \pm j572\pi = 742\pi e^{\pm j130}$

$S_{3,4} = 512\pi Cos\frac{5\pi}{8} \pm j1494\pi Sin\frac{5\pi}{8} = -196\pi \pm j1380\pi = 1394\pi e^{\pm j98}$

$$H_c(S) = \frac{(742\pi)(1394\pi)^2}{[S^2 - S*2*742\pi Cos(130) + (742\pi)^2]\ [S^2 - S*2*1394\pi Cos(98) + 1394\pi^2]}$$

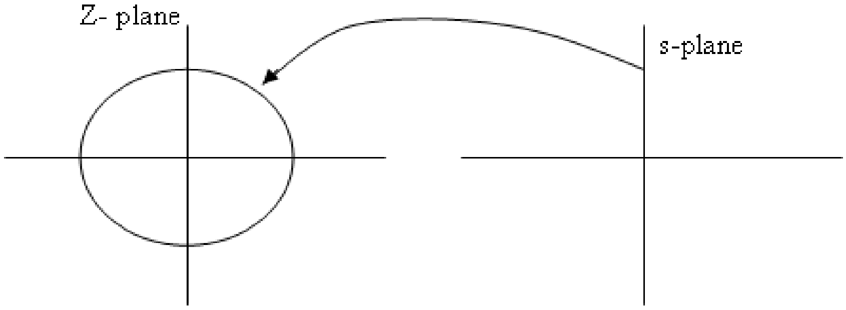
- Chebyshev filter poles are closer to the jΩ axis, therefore filter response exhibits a ripple in the pass band. There is a peak in the pass band for each pole in the filter, located approximately at the ordinate value of the pole.
- Exhibits a smaller transition region to reach the desired attenuation in the stop band, when compared to Butterworth filter.
- Phase response is similar.
- Because of proximity of Chebyshev filter poles to jΩ axis, small errors in their locations, caused by numerical round off in the computations, can results in significant changes in the magnitude response. Choosing the smaller value of μ will provide some margin for keeping the ripples within the pass band specification. However, too small a value for μ may require an increase in the filter order.
- It is reasonable to expect that if relevant zeros were included in the system function, a lower order filter can be found to satisfy the specification. These relevant zeros could serve to achieve additional attenuation in the stop band. The elliptic filter does exactly this.

7.3 IMPULSE INVARIANCE METHOD

$$H(z) = \sum_{n=0}^{\infty} h(n)Z^{-n}$$

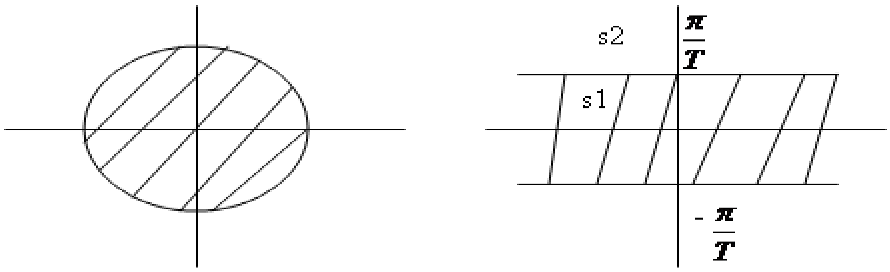
H(z) (at z = e^{ST}) = $\sum h(n)e^{-STn}$

$re^{jw} = e^{(\sigma + j\Omega)T}$ r = $e^{\sigma T}$ $e^{jw} = e^{j\Omega T}$ => $w = \Omega T$



Let S_1 = $\sigma + j\Omega$ => Z_1 = $e^{\sigma T}e^{j\Omega T}$

S_2 = $\sigma + j(\Omega + \frac{2\pi}{T})$ => Z_2 = $e^{\sigma T}e^{j\Omega T + j2\pi} = e^{\sigma T}e^{j\Omega T}$



If the real part is same, imaginary part is differ by integral multiple of 2$\frac{\pi}{T}$, this is the biggest disadvantage of Impulse Invariance method.

Let $H_A(S)$ = $\frac{s+a}{(s+a)^2+b^2}$ = $\frac{s+a}{(s+a+jb)+(s+a-jb)}$

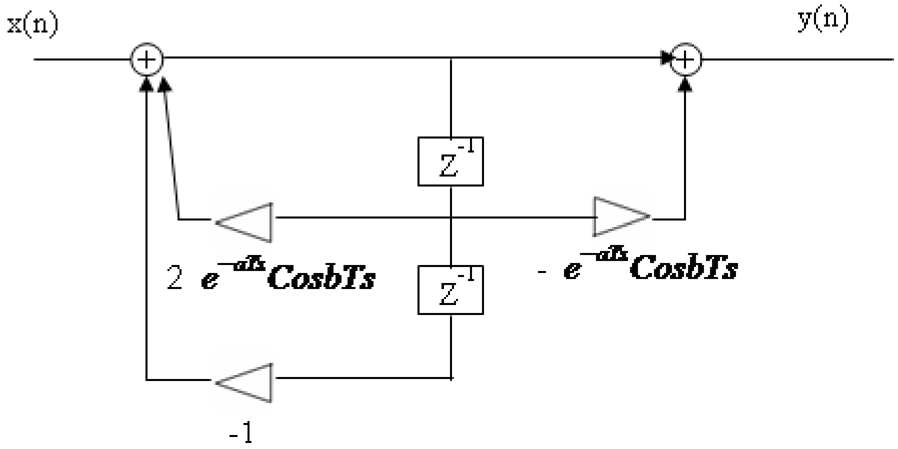
$h_A(t)$ = $e^{-at}Cosbt$ for t≥ 0 s_1 = -a-jb

 = 0 otherwise s_2 = -a+jb

h (nTs) = $e^{-anTs}Cos(bnTs)$ for n≥ 0

H(z) = $\frac{1-e^{-aTs}Cos(bTs)Z^{-1}}{1-2e^{-aTs}Cos(bTs)Z^{-1}+Z^{-2}}$ = $\frac{1-e^{-aTs}Cos(bTs)Z^{-1}}{(1-e^{-(a+jb)Ts}Z^{-1})(1-e^{-(a-jb)Ts}Z^{-1})}$

The pole located at s=p is transformed into a pole in the Z-plane at Z = e^{pTs}, however, the finite zero located in the s-plane at s= -a was not converted into a zero in the z-plane at Z = e^{-aTs}, although the zero at s=∞ was placed at z=0.



Desing a Chebyshev LPF using Impulse-Invariance Method.

$S_{1,2} = -473\pi \ \pm j\ 572\pi$

$S_{3,4} = -196\pi \ \pm j\ 1380\pi$

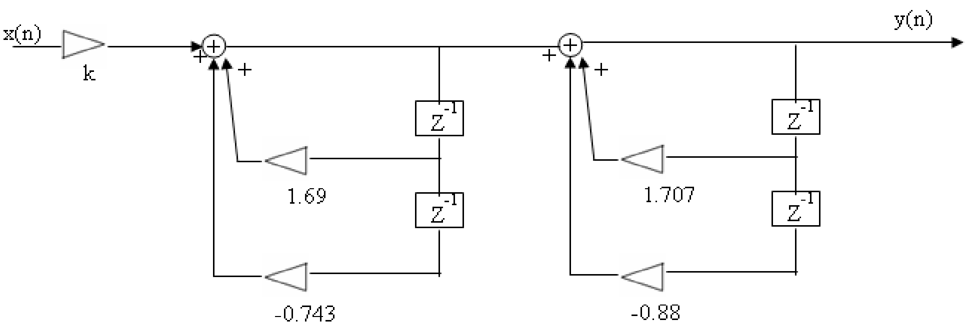
[The freq response for analog filter we plotted over freq range 0 to 10000π. To set the discrete-time freq range $(0, \frac{\pi}{Ts})$, therefore Ts = 10^{-4}]

$Z_{1,2} = e^{S_{1,2}Ts} = e^{-0.148 \pm j0.179} = 0.862\ e^{\pm j10.2}$

$Z_{3,4} = e^{S_{3,4}Ts} = e^{-0.061 \pm j0.433} = 0.94\ e^{\pm j24.8}$

$$H(z) = \frac{k}{(1-2*0.862Cos10.2Z^{-1}+0.862^2Z^{-2})(1-2*0.94Cos24.8Z^{-1}+0.94^2Z^{-2})}$$

$$= \frac{k}{(1-1.69Z^{-1}+0.743Z^{-2})(1-1.707Z^{-1}+0.88Z^{-2})}$$

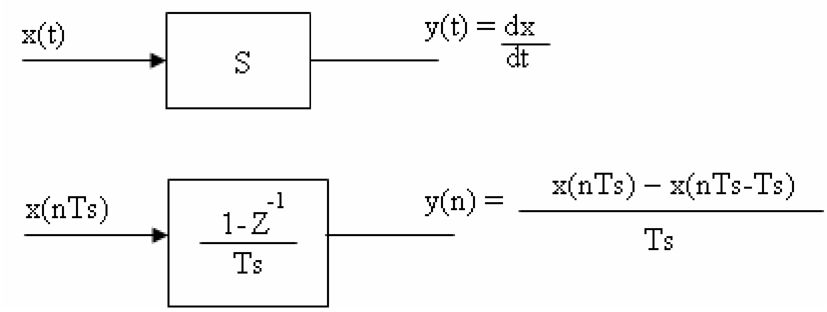


Methods to convert analog filters into Digital filters:

1. By approximation of derivatives

$$\frac{dx}{dt}/\ t=nTs \ = \frac{x(nTs)-x(nTs-Ts)}{Ts}$$

$$S = \frac{1-Z^{-1}}{Ts}$$



Or

Using forward-difference mapping based on first order approximation Z = $e^{sTs} \cong 1+STs$

$$S = \frac{Z-1}{Ts}$$

Using backward- difference mapping is based on first order approximation

$$Z^{-1} = e^{-sTs} \cong 1 - STs$$

$$S \cong \frac{Z-1}{ZTs} = \frac{1-Z^{-1}}{Ts}$$

$$\frac{d^2x}{dt^2} / t = nTs = \frac{d}{dt}\left[\frac{dx}{dt}\right] / t = nTs$$

$$= \frac{\frac{x(nTs)-x(nTs-Ts)}{Ts} - \frac{x(nTs-Ts)-x(nTs-2Ts)}{Ts}}{Ts}$$

$$= \frac{x(nTs)-2x(nTs-Ts)+x(nTs-2Ts)}{Ts^2}$$

$$S^2 = \frac{1-2Z^{-1}+Z^{-2}}{Ts^2} = \left(\frac{1-Z^{-1}}{Ts}\right)^2$$

Therefore $S^k = \left[\frac{1-Z^{-1}}{T}\right]^k$

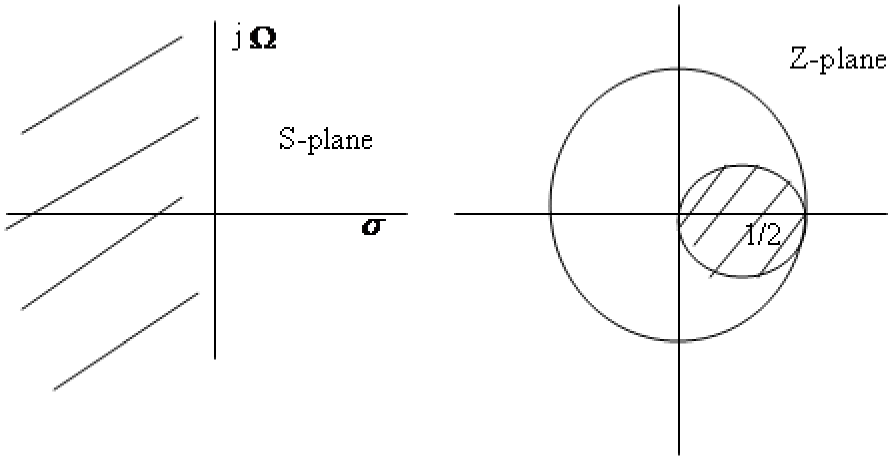
Therefore H(z) = $H_a(s)$ /s=$\left[\frac{1-Z^{-1}}{T}\right]$ using backward difference

$$Z = \frac{1}{1-STs} \qquad = 0.5 + \frac{0.5(1+STs)}{1-STs}$$

$$= \frac{1}{1-j\Omega Ts} = \frac{1}{1+\Omega^2 Ts^2} + \frac{j\Omega Ts}{1+\Omega^2 Ts^2}$$

Z - 0.5 = $\frac{0.5(1+STs)}{(1-STs)}$

$|z-0.5| = 0.5$ is mapped into a circle of radius 0.5, centered at Z=0.5



Using Forward-difference

S= $\frac{Z-1}{Ts}$ Z=1+STs

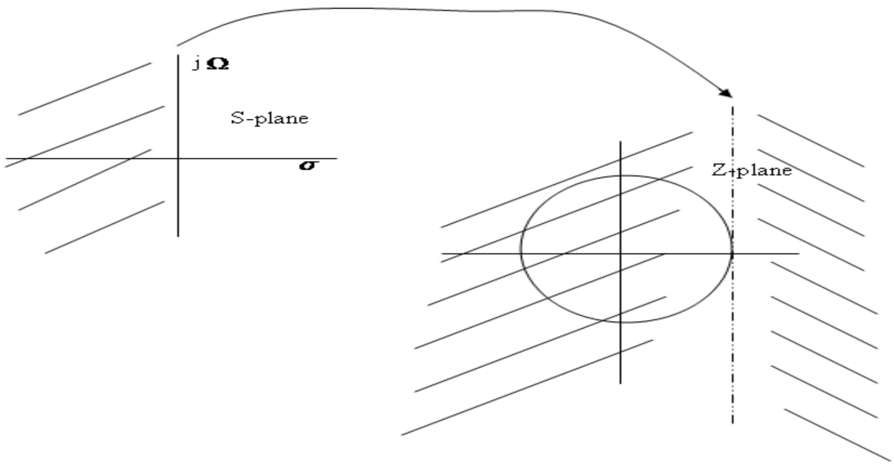
u+jv = 1+ ($\sigma + j\Omega$) Ts

if σ=0 u=1 and jΩ axis maps to Z=1

If σ>0, then u>1, the RHS-plane maps to right of z=1.

If σ<0, then u<1, the LHS-plane maps to left of z=1.

The stable analog filter may be unstable digital filter.



7.4 Bilinear Transformation

- Provides a non linear one to one mapping of the frequency points on the jw axis in s-plane to those on the unit circle in the z-plane.

- This procedure also allows us to implement digital HP filters from their analog counter parts.

$$S = \frac{2}{Ts}\frac{Z-1}{Z+1} = \frac{2}{Ts}\frac{1-Z^{-1}}{1+Z^{-1}}$$

{Using trapezoidal rule $y(n)=y(n-1)+0.5T_s[x(n)+x(n-1)]$

$H(Z)=2(Z-1) / [T_s(Z+1)]$ }

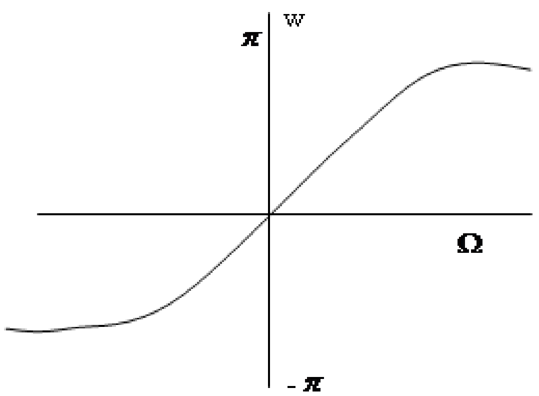
To find H(z), each occurrence of S in $H_A(s)$ is replaced by $\frac{2}{Ts}\frac{Z-1}{Z+1}$

And $Z = \dfrac{\frac{STs}{2}+1}{\frac{STs}{2}-1}$

$$e^{jw} = \frac{j\Omega\frac{Ts}{2}+1}{j\Omega\frac{Ts}{2}-1} = \frac{\left(\Omega^2\left(\frac{Ts}{2}\right)^2+1\right)^{1/2} e^{j\tan^{-1}\Omega\frac{Ts}{2}}}{\left(\Omega^2\left(\frac{Ts}{2}\right)^2+1\right)^{1/2} e^{j\tan^{-1}-\Omega\frac{Ts}{2}}}$$

$$e^{jw} = e^{j2\tan^{-1}\Omega\frac{Ts}{2}} \qquad w=2\tan^{-1}\frac{\Omega Ts}{2}$$

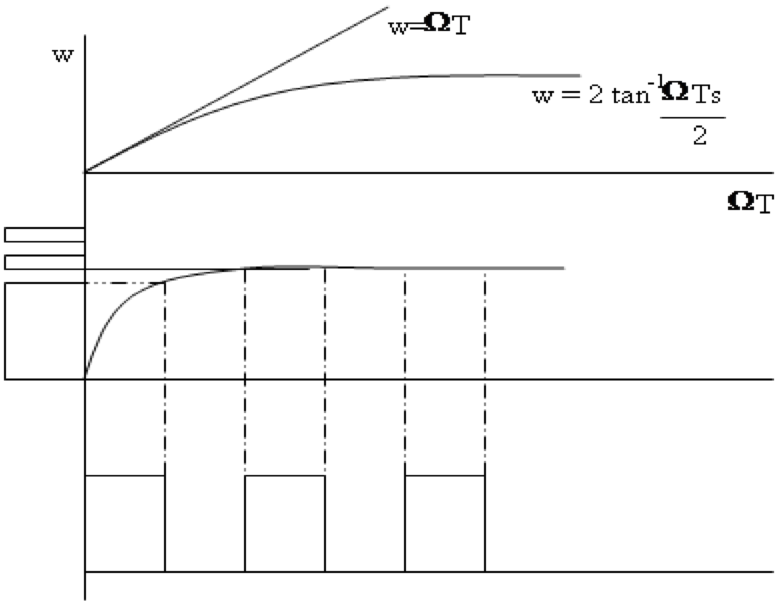
The entire $j\Omega$ axis in the s-plane $-\infty<j\Omega<\infty$ maps exactly once onto the unit circle $-\pi<w\le\pi$ such that there is a one to one correspondence between the continuous-time and discrete time frequency points. It is this one to one mapping that allows analog HPF to be implemented in digital filter form.



As in the impulse invariance method, the left half of s-plane maps on to the inside of the unit circle in the z-plane and the right half of s-plane maps onto the outside.

In Inverse relationship is $\Omega = \frac{2}{Ts}\tan\left(\frac{w}{2}\right)$

For smaller value of frequency $\Omega = \frac{2}{Ts}\frac{Sin\frac{w}{2}}{Cos\frac{w}{2}} = \frac{2}{Ts}\frac{\left(\frac{w}{2}-\frac{w^3}{8}+....\right)}{1-\frac{w^2}{4}+....} = \frac{w}{Ts}$



(B.W of higher freq pass band will tend to reduce disproportionately)

The mapping is $\cong$ linear for small Ω and w. For larger freq values, the non linear compression that occurs in the mapping of Ω to w is more apparent. This compression causes the transfer function at the high Ω freq to be highly distorted when it is translated to the w-domain.

Prewarping Procedure:

When the desired magnitude response is piece wise constant over frequency, this compression can be compensated by introducing a suitable prescaling or prewarping to the Ω freq scale. Ω scale is converted into Ω^* scale.

$$\Omega^* = \frac{2}{Ts}\tan\left(\frac{\Omega Ts}{2}\right)$$

We now derive the rule by which the poles are mapped from the s-plane to the z-plane.

Let $H_A(s) = \frac{1}{S-Sp}$ $\qquad$ $S=S_p$

$$H(z) = \frac{1}{\frac{2}{Ts}\left(\frac{1-Z^{-1}}{1+Z^{-1}}\right)-Sp} = \frac{Ts(1+Z^{-1})}{(2-SpTs)\left(1-\frac{2+SpTs}{2-SpTs}Z^{-1}\right)}$$

A pole at $S=S_p$ in the s-plane gets mapped into a zero at z= -1 and a pole at Z = $\frac{2+SpTs}{2-SpTs}$

Ex:

Chebyshev LPF design using the Bilinear Transformation

Pass band:

$-1<|H(j\Omega)|$dB $\leq$ 0 for $0 \leq \Omega \leq 1404\pi$ =4411 rad

Stop band:

$|H(j\Omega)|$ dB < -60 for $\Omega \geq 8268\pi$ rad/sec =25975 rad/s

Let the Ts = 10^{-4} sec

Prewarping values are

$$\Omega_p^* = \frac{2}{Ts}\tan\left(\frac{\Omega Ts}{2}\right) = 2*10^4 \tan(0.0702\pi) = 4484 \text{ rad/sec}$$

And $$\Omega_s^* = \frac{2}{Ts}\tan\left(\frac{\Omega Ts}{2}\right) = 2*10^4 \tan(0.4134\pi) = 71690 \text{ rad/sec}$$

The modified specifications are

Pass band:

$-1<|H(j\Omega^*)|$dB $\leq$ 0 for $0 \leq \Omega^* \leq 4484$ rad/s

Stop band:

$|H(j\Omega^*)|$ dB < -60 for $\Omega^* \geq 71690$rad/sec

Value of μ: is determined from the pass band ripple $10\log\left(1+\mu^{-2}\right)^{-1} > -1dB$

$$\mu = 0.508$$

Value of N: is determined from

$$|H_c(j\Omega s^*)|^2 = \left[1+\mu^2 C_N^2\left(\frac{\Omega s^*}{\Omega p^*}\right)\right]^{-1} < 10^{-6}$$

Since $\frac{\Omega s^*}{\Omega p^*} = 16$

$C_N^2(16) < \left[\frac{(10^6-1)}{\mu^2}\right]$

$C_N(16) < \left[\frac{(10^6-1)}{(0.508)^2}\right]^{\frac{1}{2}} = 1969$

$C_3(16) = 16301$

N = 3 is sufficient

Using Impulse Invariance method a value of N=4 was required.

$\rho = 4.17$

Major R $= \Omega p * \dfrac{\left(\rho^{\frac{1}{N}} + \rho^{\frac{-1}{N}}\right)}{2} = \dfrac{4484}{2}\left(4.17^{\frac{1}{3}} + 4.17^{-\frac{1}{3}}\right) = 5001$

r = $\dfrac{4484}{2}\left(4.17^{\frac{1}{3}} - 4.17^{-\frac{1}{3}}\right) = 2216$

Since there are three poles, the angles are $\pi \,\&\, \dfrac{2\pi}{3}$

S_1 = r cosθ + j Rsinθ = -2216

$S_{2,3}$ = 2216 Cos$\dfrac{2\pi}{3}$ $\pm$j 5001 Sin$\dfrac{2\pi}{3}$ = -1108$\pm$j 4331 = 4470 $e^{\pm j104.4}$

$H_c(s) = \dfrac{4.43*10^{10}}{(s+2216)(S^2+2223s+4470^2)}$

Pole Mapping

At $S=S_1$

In the Z-plane there is zero at Z = -1 and pole at Z = $\dfrac{2+(-2216*10^{-4})}{2-(-2216*10^{-4})} = 0.801$

S2,3 = there are two zeros at Z=-1

Z = $\dfrac{2+(-1108 \pm j4331)*10^{-4}}{2-(-1108 \pm j4331)*10^{-4}} = 0.801 \pm j0.373 = 0.9e^{\pm j24.5}$

H(z) = 4.29 * 10^{-3} $\dfrac{1+Z^{-1}}{1-0.801Z^{-1}} \dfrac{1+2Z^{-1}+Z^{-2}}{1-1.638Z^{-1}+0.81Z^{-2}}$

Pole Mapping Rules:

$H_z(z) = 1-CZ^{-1}$ zero at Z=C and pole at Z = 0

$H_p(z) = \frac{1}{1-dZ^{-1}}$ pole ar Z=d and zero at z=0

C and d can be complex-valued number.

<u>Pole Mapping for Low-Pass to Low Pass Filters</u>

Applying low pass to low pass transformation to $H_z(z)$ α we get

$$H_{LZ}(Z) = 1\text{-}c\frac{Z^{-1}-\alpha}{1-\alpha Z^{-1}} = (1+c\alpha)\ \frac{1-\left(\frac{\alpha+c}{1+c\alpha}\right)Z^{-1}}{1-\alpha Z^{-1}}$$

The low pass zero at z=c is transformed into a zero at z=C1 where C1 = $\left(\frac{\alpha+c}{1+c\alpha}\right)$

And pole at z=0 is Z=α

Similarly,

$$H_{LP}(Z)= \frac{1-\alpha Z^{-1}}{(1+d\alpha)\left[1-\left(\frac{\alpha+d}{1+\alpha d}\right)Z^{-1}\right]}$$

Pole at z=d => Z= $\left(\frac{\alpha+d}{1+\alpha d}\right)$

Zero at z=0 => z = α

$$H(z) = K\ \frac{(1+Z^{-1})(1+2Z^{-1}+2Z^{-2})}{(1-0.622Z^{-1})(1-1.07Z^{-1}+0.674Z^{-2})}$$

$$K = \frac{[1+(-1)(-0.356)]^3}{(1+0.801*-0.356)(1+(0.819+j0.373)(-0.356))(1+(0.819-j0.373)(-0.356))} = 0.029$$

8.FIR Filters

Phase Delay: $\tau_p = \frac{\theta(\Omega)}{\Omega}$

Group Delay: $\tau_g = \frac{-d\theta(\Omega)}{d\Omega}$

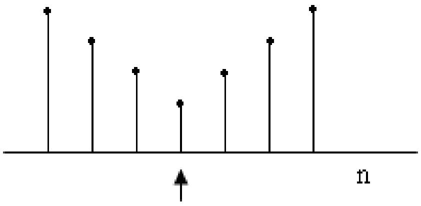
If $\tau_p = \tau_g$ =constant and independent of frequency are called as constant time delay or linear phase filters.

$$\theta(\Omega) = \theta o - \tau\Omega$$

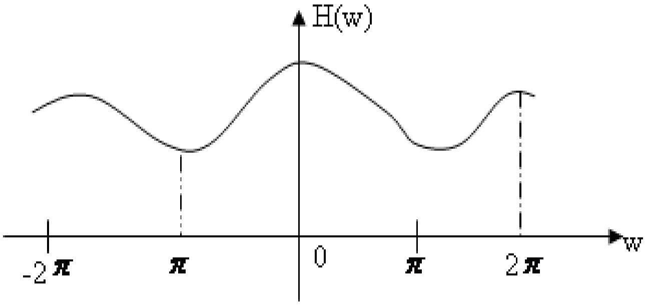
$$\tau_p = \frac{\theta o}{\Omega} - \tau \qquad \text{Changes with frequency}$$

$\tau_g = -\tau$ =constant.

8.1 Type 1 Sequence



Odd length, even symmetry



Center of Symmetry M= $\frac{N-1}{2}$ = integer value

$$H(e^{j\Omega T}) = \sum_{n=0}^{(\frac{N-3}{2})} h(n)e^{-j\Omega nT} + h(\frac{N-1}{2})\, e^{-j\Omega T(\frac{N-1}{2})} + \sum_{n=\frac{N+1}{2}}^{(N-1)} h(n)e^{-j\Omega nT}$$

Let N-1-n =n

$$\sum_{n=0}^{(N-3)/2} h(n)e^{-j\Omega nT} + h(\frac{N-1}{2})\, e^{-j\Omega T(\frac{N-1}{2})} + \sum_{n=0}^{(\frac{N-3}{2})} h(N-1-n)e^{-j\Omega(N-1-n)T}$$

$$\sum_{n=0}^{(N-3)/2} h(n)e^{-j\Omega nT} + h(\frac{N-1}{2})\, e^{-j\Omega T(\frac{N-1}{2})} + \sum_{n=0}^{(\frac{N-3}{2})} h(n)e^{-j\Omega(N-1-n)T}$$

$$e^{-j\Omega\frac{N-1}{2}T}\left[\sum_{n=0}^{(N-3)/2} h(n)e^{j\Omega T[\frac{N-1}{2}-n]} + h(\frac{N-1}{2}) + \sum_{n=0}^{(\frac{N-3}{2})} h(n)e^{-j\Omega(\frac{N-1}{2}-n)T}\right.$$

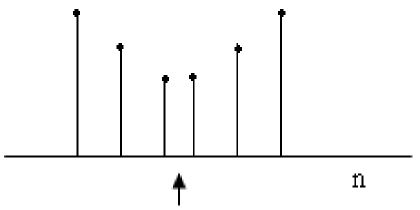
$$H(e^{j\Omega T}) = e^{-j\Omega\frac{N-1}{2}T}\left[\sum_{n=0}^{(N-3)/2} 2h(n)cos\Omega T(\frac{N-1}{2}-n) + h(\frac{N-1}{2})\right]$$

$$H(w) = \left[h(M) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)Cos\Omega T(n-M)\right]e^{-j\Omega MT}$$

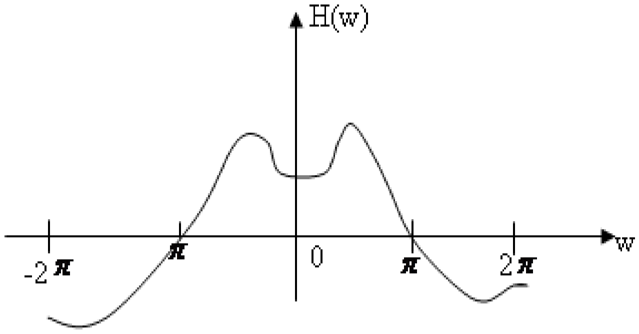
$$\tau = \left(\frac{N-1}{2}\right)T$$

Amplitude spectrum is even symmetric about w=0 & π & both H(0) & H(π) can be non zero.

8.2 Type 2 Sequence



N = Even length, even symmetry



h(n) = h(N-1-n)

Center of Symmetry M= $\frac{N-1}{2}$ = half-integer value

$$H(e^{j\Omega T}) = \sum_{n=0}^{N-1} h(n)e^{-j\Omega nT}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\Omega nT} + \sum_{n=\frac{N}{2}}^{N-1} h(n)e^{-j\Omega nT}$$

Let N-1-n = m

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\Omega nT} + \sum_{m=\frac{N}{2}-1}^{0} h(N-1-m)e^{-j\Omega T(N-1-m)}$$

But h(N-1-m) = h(m)

$$== \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\Omega nT} + \sum_{m=0}^{\frac{N}{2}-1} h(m)e^{-j\Omega T(N-1-m)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\Omega nT} e^{\pm j\Omega T\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N}{2}-1} h(m)e^{-j\Omega T(N-1-n)} e^{\pm j\Omega T\left(\frac{N-1}{2}\right)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} 2h(n)e^{-j\Omega T\left(\frac{N-1}{2}\right)} \left[\frac{e^{-j(\Omega nT-\frac{\Omega T}{2}(N-1)} + e^{-j(\Omega T(N-1-n)-\frac{\Omega T}{2}(N-1)}}{2}\right]$$

$$= \sum_{n=0}^{\frac{N}{2}-1} 2h(n)e^{-j\Omega T\left(\frac{N-1}{2}\right)} \cos\Omega T\left(n-\left(\frac{N-1}{2}\right)\right)$$

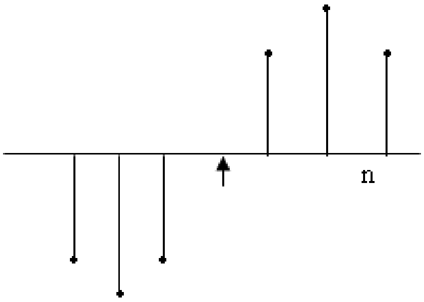
$$= e^{-j\Omega T\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} 2h(n)\cos\Omega T\left(n-\left(\frac{N-1}{2}\right)\right)$$ ----Magnitude

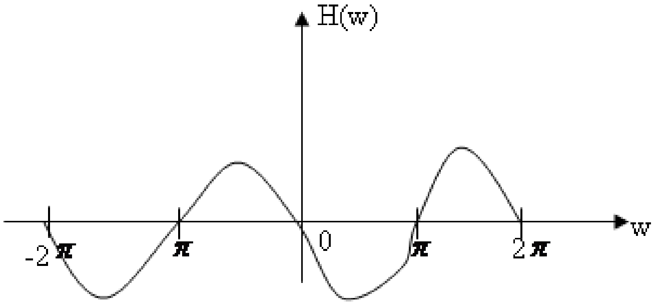
$\tau = \left(\frac{N-1}{2}\right)T$ Linear Phase

H(w) = $\left[2\sum_{n=0}^{\frac{N}{2}-1} h(n) Cos\Omega T(n-M)\right] e^{-j\Omega MT}$

The Amplitude spectrum is even symmetric about w=0 & odd symmetric about w= π & both H(π) is always zero for type 1 & 2 : Constant phase delay and group delay.

8.3 Type 3 Sequence





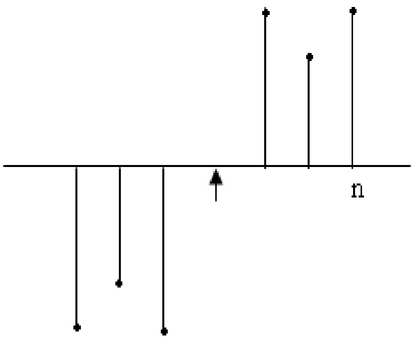
N = Odd, Odd symmetry

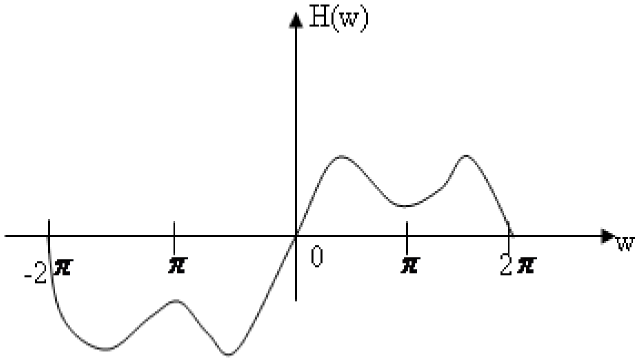
M= $\frac{N-1}{2}=$ integer value

$$H(w) = j\left[2\sum_{n=0}^{\frac{N-3}{2}} h(n) Sin\Omega T(M-n)\right]e^{-j\Omega MT}$$

It shows generalized linear phase of $\frac{\pi}{2}-\Omega MT$ and constant group delay of M. The Amplitude spectrum is odd symmetric about w=0 & w=π and H(0) & H(π) are always zero. (Generalized means $\theta(\Omega)$ may jump of π at $\Omega=0$ if H(e^{jw}) is imaginary.

8.4 Type 4 Sequence





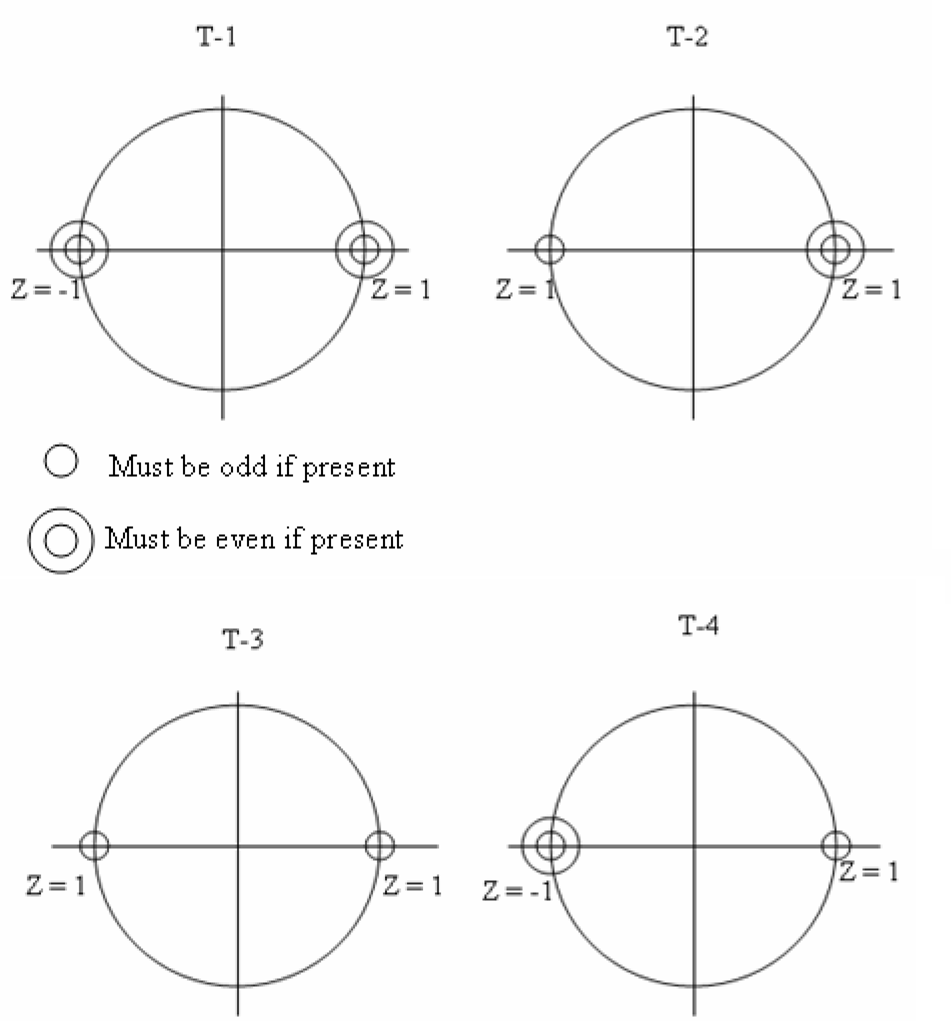
N = even, Odd symmetry

$$H(w) = j\left[2\sum_{n=0}^{\frac{N}{2}-1} h(n) Sin\Omega T(M-n)\right]e^{-j\Omega MT}$$

Generalized linear phase and constant group delay of M. The Amplitude spectrum is odd symmetric about w=0 & even symmetric about w=π and H(0)=0 always.

8.5 Poles & Zeros of linear phase sequences:

The poles of any finite-length sequence must lie at z=0. The zeros of linear phase sequence must occur in conjugate reciprocal pairs. Real zeros at z=1 or z=-1 need not be paired (they form their own reciprocals), but all other real zeros must be paired with their reciprocals. Complex zeros on the unit circle must be paired with their conjugate (that form their reciprocals) and complex zeros anywhere else must occur in conjugate reciprocal quadruples. To identify the type of sequence from its pole-zero plot, all we need to do is check for the presence of zeros at z= $\pm$ and count their number. A type-2 seq must have an odd number of zeros at z=-1, a type-3 seq must have an odd number of zeros at z=-1 and z=1, and type-4 seq must have an odd number of zeros at z=1. The no. of other zeros if present (at z=1 for type=1 and type-2 or z=-1 for type-1 or type-4) must be even.



Prove:

H(Z) = $\sum_{n=0}^{N-1} h(n)Z^{-n}$

H(Z)$_{at\ Z=Z0}$ = H(Z$_0$)= $\sum_{n=0}^{N-1} h(n)\ Z_0^{-n}$

=h(0) + h(1) Z_0^{-1} ++ h(2) Z_0^{-2} + …….. + h(N-1) $Z_0^{-(N-1)}$=0

For linear phase h(N-1-n)=h(n)

h(N-1) + h(N-2) Z_0^{-1} ++ h(N-2) Z_0^{-2} + …….. + h(0) $Z_0^{-(N-1)}$=0

$Z_0^{-(N-1)}$[h(0) + h(1) Z_0^{1} ++ h(2) Z_0^{2} + …….. + h(N-1) $Z_0^{(N-1)}$]=0

$Z_0^{-(N-1)}$= $\sum_{n=0}^{N-1} h(n)(Z_0^{-}\ )^{-n}$=0

Therefore H(Z$_0$)= H(Z_0^{-1})=0

If Z$_0$ is a zero of H(Z), then Z_0^{-1} is also a zero

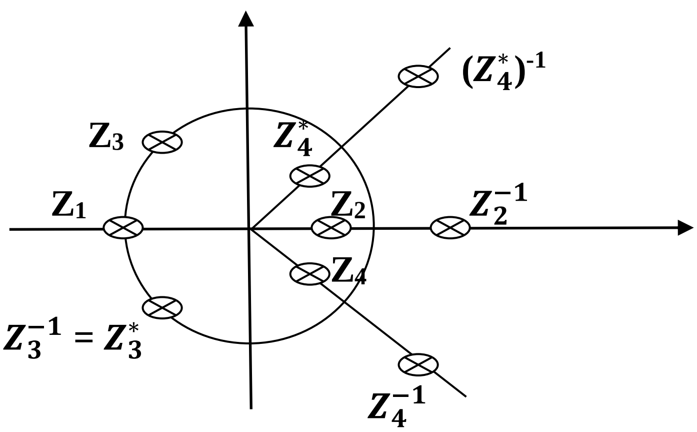
1 If Z$_1$ = -1 then Z_1^{-1} = Z$_1$, then the zero lie at Z$_1$=-1

This group contains only one zero on the unit circle

2) If Z$_2$ is real zero with |Z$_2$| <1 then Z_2^{-1} is also a real zero and there are two zeros in this group

3) If Z$_3$ is a comple zero with |Z$_3$| =1 then Z_3^{-1} = Z_3^{*} and there are two zeros in this group

4) If Z$_4$ is a complex zero with |Z$_4$| ≠1 then this group contain four zeros Z$_4$, Z_4^{-1} = Z_4^{*} , (Z_4^{*})$^{-1}$



FIR Filters

8.6 Fourier series Method

$$\frac{-Fs}{2} \le F \le \frac{Fs}{2}$$

$$\frac{-2\pi Fs}{2} \le 2\pi F \le \frac{2\pi Fs}{2}$$

$$\frac{-\Omega s}{2} \le \Omega \le \frac{\Omega s}{2}$$

1. Frequency response of a discrete-time filter is a periodi function with period Ωs (sampling freq).

2. From the F.S analysis we know that any periodic function can be expressed as a linear combination of complex exponentials.

Therefore desired freqency response of a discrete time filter can be represented by F.S as

$$H(e^{j\Omega T}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\Omega nT}$$ T = sampling period

The F.S co-efficient or impulse response samples of filter can be obtained using

h (n) = $\dfrac{1}{\Omega s} \displaystyle\int_{-\Omega s/2}^{\Omega s/2} H(e^{j\Omega T}) e^{j\Omega nT} d\Omega$

clearly if we wish to realize this filter with impulse response h(n), then it must have finite no. of co-efficient, which is equivalent to truncating the infinite expansion of $H(e^{j\Omega T})$, which leads to approximation of $H(e^{j\Omega T})$, which is denoted by

$$H_1(e^{j\Omega T}) = \sum_{n=-M}^{M} h(n) e^{-j\Omega nT} .$$

We choose M= $\dfrac{N-1}{2}$, in order to keep 'N' no of samples in h(n).

H1(z) = $\displaystyle\sum_{n=-M}^{M} h(n) Z^{-n}$

However, this filter can't be physically realizable due to the presence of +ve powers of Z, means that the filter must produce an output that is advanced in time with respect to the i/p. This difficulty can be overcome by introducing a delay M= $\frac{N-1}{2}$ samples.

Therefore $\quad H(z) = Z^{-M} H_1(z) = Z^{-M} \sum_{n=-M}^{M} h(n) Z^{-n}$

$H(z) = h(-M)Z^0 + h(-M+1) Z^{-1} + \ldots + h(M) Z^{-2M}$

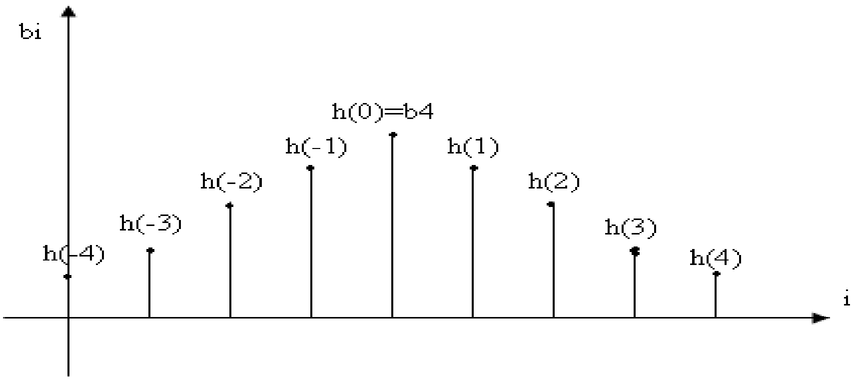
Let $b_i = h(i-M)$ i=0 to 2M

$H(z) = \sum_{i=0}^{2M} b_i Z^{-i}$ be the transfer function of discrete filter that is physically realizable.

Properties:

1. N=2M+1, impulse response co-eff, bi = 0 to 2M.
2. h(n) is symmetric about b_M

Ex: M=4



3. The duration of impulse response is Ti = 2MT
4. Its magnitude and time delay function can be found in the following way

$$H(e^{j\Omega T}) = e^{-j\Omega MT} H_1(e^{j\Omega T})$$

$$\left|H(e^{j\Omega T})\right| = \left|H_1(e^{j\Omega T})\right|$$

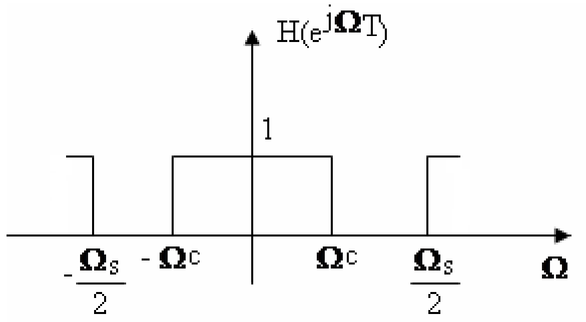
This implies that magnitude response of the filter we have desired approximates the desire magnitude response. The time delay of $H(e^{jw})$ is a constant M. thus sinusoids of different frequencies are delayed by the same amount as they are processed by the filter, we have designed. Consequently, this is a linear phase filter, which means that it does not introduce phase distortion.

Ex:

Design a LPF (FIR) filter with frequency response

$H(e^{j\Omega T}) = 1$ for $|\Omega| \leq \Omega c$

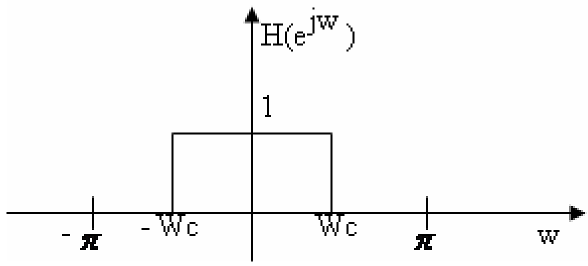
$= 0$ for $\Omega c < \Omega < \frac{\Omega s}{2}$



$$h(n) = \frac{1}{\Omega s}\int_{-\Omega c}^{\Omega c} e^{j\Omega nT} d\Omega$$

$$= \frac{2}{\Omega s}\int_{0}^{\Omega c} Cos(\Omega nT) d\Omega$$

$$= \frac{2}{\Omega s}\frac{Sin\Omega cnT}{nT}$$



$$= \frac{2}{2\pi Fsn.\frac{1}{Fs}} Sin\Omega cnT = \frac{1}{n\pi} Sin\Omega cnT$$

$$w = \Omega T = \frac{\Omega s}{2} T = \frac{2\pi Fs}{2}\frac{1}{Fs} = \pi$$

bi = h(i-M)

$$H(z) = \sum_{i=0}^{2M} b_i Z^{-i}$$

Ex:

Design LPF that approximate following freq response.

H(F) = 1 $0 \leq F \leq 1000Hz$

= 0 else where $1000 \leq F \leq Fs/2$

When the sampling frequency is 8000 SPS. The impulse response duration is to be limited to 2.5ms

Ti = 2MT

M = $\dfrac{2.5*10^{-3}}{2*\frac{1}{800}} = 10$ N=21

h(n) = $\dfrac{1}{\Omega s}\int_{-\Omega c}^{\Omega c} 1.e^{j\Omega nT} d\Omega$

$= \dfrac{1}{2\pi Fs}\int_{-Fc}^{Fc} 1.e^{j2\pi FnT}\, 2\pi dF \qquad = \dfrac{1}{Fs}\int_{-Fc}^{Fc} e^{j2\pi FnT} dF = \dfrac{2}{Fs}\int_{0}^{Fc} Cos(2\pi FnT)dF$

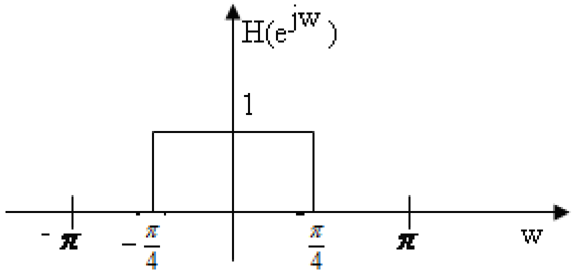
$= \dfrac{1}{n\pi} Sin 2\pi FcnT = \dfrac{1}{n\pi} Sin(0.25n\pi)$

OR

w = ΩT = $2\pi * 1000 * \dfrac{1}{8000} = \dfrac{\pi}{4}$

H_c(w) = 1 $\quad |w| \le \pi/4$

= 0 else where



$$\frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} 1.e^{jwn} dw = \frac{1}{n\pi} Sin(0.25n\pi)$$

h(0) = 0.25 h(6) = -0.05305

h(1) = 0.22508 h(7) = -0.03215

h(2) = 0.15915 h(8) = 0

h(3) = 0.07503 h(9) = 0.02501

h(4) = 0 h(10) = 0.03183

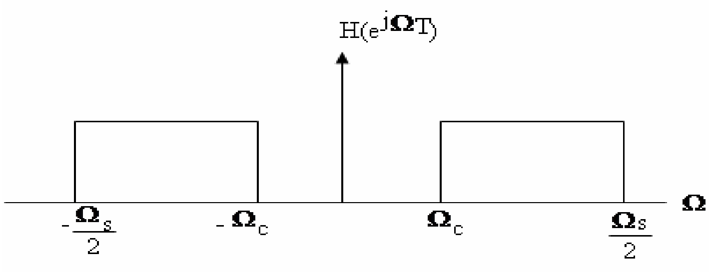
h(5) = -0.04502

bi = h(i-10)

H(z) = $\sum_{i=0}^{20} b_i Z^{-i}$

FIR HPF

$$h(n) = \frac{1}{\Omega s}\left[\int_{-\Omega s/2}^{-\Omega c} 1.e^{j\Omega nT} d\Omega + \int_{\Omega c}^{\Omega s/2} e^{j\Omega nT} d\Omega\right]$$



$$= \frac{1}{\Omega s}\left[\frac{e^{j\Omega nT}}{jnT}\Big|_{-\Omega s/2}^{\Omega c} + \frac{e^{j\Omega nT}}{jnT}\Big|_{-\Omega c}^{\Omega s/2}\right]$$

$$= \frac{1}{\Omega s}\left[\frac{e^{-j\Omega cnT} - e^{-j\frac{\Omega s}{2}nT} + e^{j\frac{\Omega s}{2}nT} - e^{j\Omega cnT}}{jnT}\right]$$

$$= \frac{-2}{\Omega s}\frac{1}{nT}\left[\frac{e^{j\Omega cnT} - e^{-j\Omega cnT}}{2j} + \frac{e^{j\frac{\Omega s}{2}nT} - e^{-j\frac{\Omega s}{2}nT}}{2j}\right]$$

$$= \frac{-2}{2\pi FsnT}\left[Sin\Omega_c nT + Sin\frac{\Omega_s nT}{2}\right]$$

$$= \frac{-1}{n\pi}[\sin\Omega_c nT + Sin\pi n] \quad = \quad \frac{-1}{n\pi}[\sin\Omega_c nT]$$

FIR BPF

$$h(n) = \frac{2}{\Omega s}\int_{\Omega l}^{\Omega u} \cos n\Omega T \quad d\Omega = \frac{1}{n\pi}[\sin\Omega_u nT - \sin\Omega_l nT]$$

Ex:

Desing a BPF for H(f) = 1 $160 \le F \le 200$Hz

= 0 else where

Fs = 800SPS

Ti = 20 ms

$$M = \frac{Ti}{2T} = \frac{20*10^{-3}}{2*\frac{1}{800}} = 8 \qquad N = 17$$

$$h(n) = \frac{1}{n\pi}\left[Sin2\pi F_u nT - Sin2\pi F_l nT\right] = \frac{\sin 0.5n\pi - \sin 0.4n\pi}{n\pi}$$

h(0) = 0.1 h(4) = 0.07568

h(1) = 0.01558 h(5) = 0.06366

h(2) = -0.09355 h(6) = -0.05046

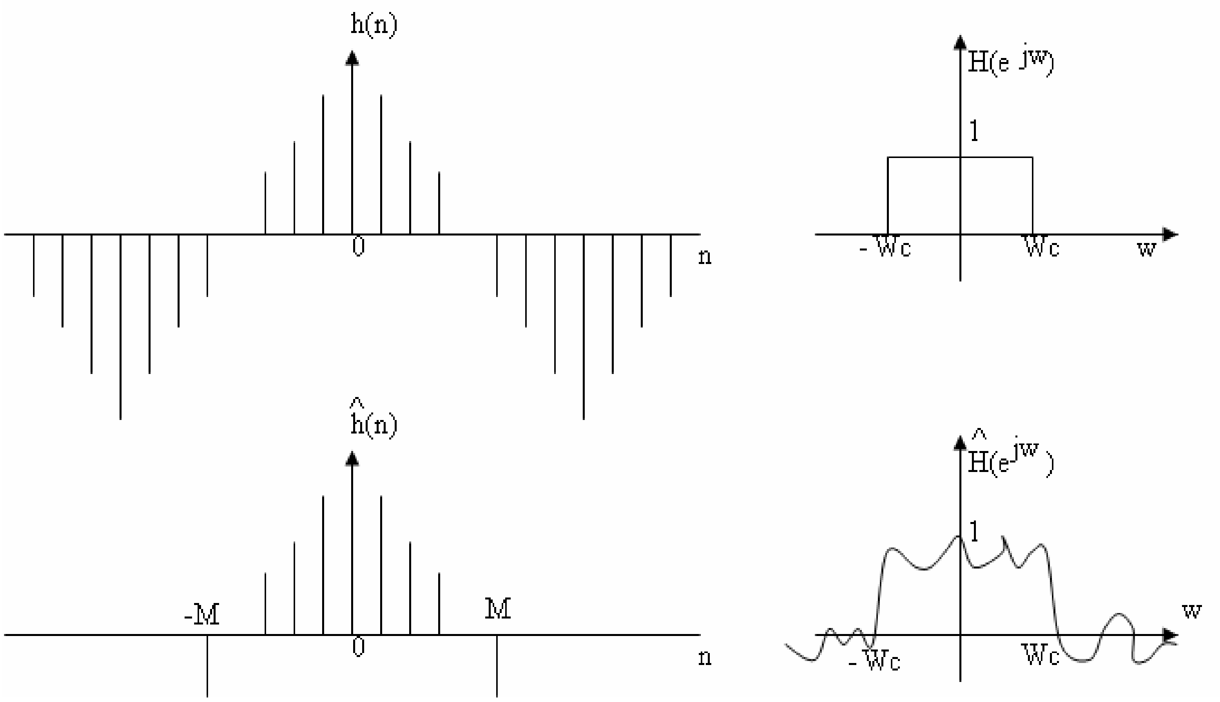
h(3) = -0.04374 h(7) = -0.07220 h(8) = 0.02338

$$H(z) = \sum_{i=0}^{16} b_i Z^{-i}$$

bi = h(i-8) h(-n) = h(n)

8.7 WINDOWING

Disadvantage of F.S is abrupt truncation of FS expansion of the freq response. This truncation result in a poor convergence of the series.



The abrupt truncation of infinite series is equivalent to multiplying it with the rectangular sequence.

$W_R(n) = 1 \qquad |n| \le M$

$\qquad = 0 \qquad$ else where

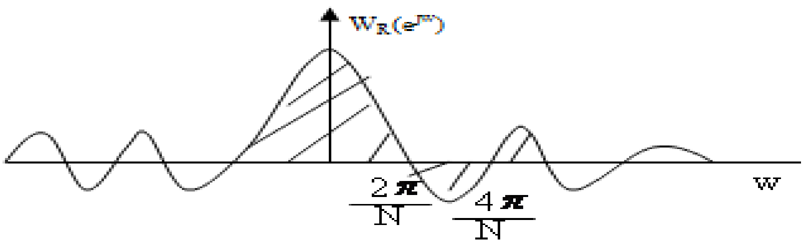
$$\hat{h}(n) = h(n)W_R(n)$$

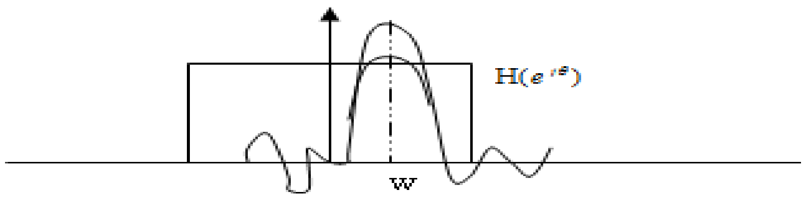
$$\hat{H}(e^{jw}) = H(e^{jw}) * W_R(e^{jw})$$

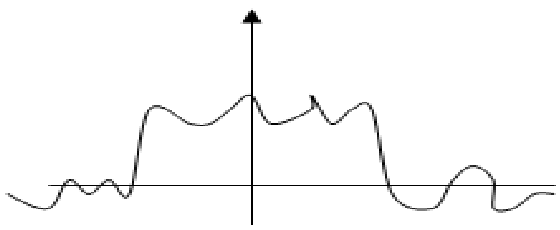
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\theta})W_R(e^{j(w-\theta)})d\theta$$

$W_R(e^{jw})$ => FT of Rectangular Window

$$W_R(e^{jw}) = \sum_{n=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} 1.e^{-jwn}dw = \frac{Sin\frac{wN}{2}}{Sin\frac{w}{2}} = \frac{NSa\frac{wN}{2}}{Sa\frac{w}{2}}$$







➢ Main lobe width = $\frac{4\pi}{N}$ & it can be reduced by increasing N, but area of side lobe will be constant.

➢ For larger value of N, transition region can be reduced, but we will find overshoots & undershoots on pass band and non zero response in stop band because of larger side lobes. So these overshoots and leakage will not change significantly when rectangular window is used. This result is known as Gibbs Phenomenon.

The desined window chts are

1. Small width of main lobe of the frequency response of the window containing as much as of the total energy as possible.
2. Side lobes of the frequency response that decrease in energy as w tends to π.
3. even function about n=0

4. zero in the range $|n| \geq \frac{N-1}{2}$

Let us consider the effect of tapering the rectangular window sequence linearly from the middle to the ends.

Triangular Window:

$$W_T(n) = 1 - \frac{2|n|}{N-1} \qquad |n| \leq \frac{N-1}{2}$$

= 0 else where

In this side lobe level is smaller than that of rectangular window, being reduced from -13 to -25dB to the maximum. However, the main lobe width is now $\frac{8\pi}{N}$. There is a trade off between main lobe width and side lobe levels.

General raised cosine window is

$$W(n) = \alpha + (1-\alpha) Cos\left(\frac{2\pi n}{N-1}\right) \qquad \text{for } |n| \leq \frac{N-1}{2}$$

= 0 else where

If α =0.5 **Hanning Window**

If α =0.54 **Hamming Window**

$$W_B(n) = 0.42 + 0.5\, Cos\left(\frac{2\pi n}{N-1}\right) + 0.08 Cos\left(\frac{4\pi n}{N-1}\right)$$ **Blackman Window**

Kaiser Window

$$W_k(n) = \frac{Io\left[\beta\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right]}{Io(\beta)} \qquad \text{for } |n| \leq \frac{N-1}{2}$$

= 0 else where

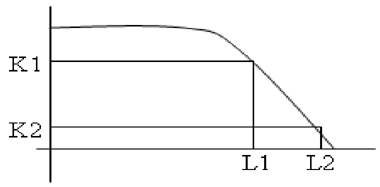
β is constant that specifies a freq response trade off between the peak height of the side lobe ripples and the width or energy of main lobe and Io(x) is the zeroth order modified Bessel function of the first kind. Io(x) can be computed from its power series expansion given by

$$Io(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

$$= 1 + \frac{0.25x^2}{(1!)^2} + \frac{(0.25x^2)^2}{(2\ !)^2} + \frac{(0.25x^2)^3}{(3\ !)^2} + \ldots..$$

Window	Peak amplitude of side lobe dB	Transition width of main lobe	Minimum stop band deviation dB
Rectangular	-13	$\frac{4\pi}{N}$ k=1	-21
Triangular	-25	$\frac{8\pi}{N}$ k=2	-25
Hanning	-31	$\frac{8\pi}{N}$ k=2	-44
Hamming	-41	$\frac{8\pi}{N}$ k=2	-53
BlackMan	-57	$\frac{12\pi}{N}$ k=3	-74
Kaiser	variable	variable	-

If we let K_1,W_1 and K_2,W_2 represent cutoff (pass band) * stop band requirements for the digital filter, we can use the following steps in design procedure.



1. Select the window type from table to be the one highest up one list such that the stop band gain exceeds K_2.
2. Select no. of points in the windows function to satisfy the transition width for the type of window used. If Wt is the transition width, we must have $Wt = W_2 - W_1 \geq k.\frac{2\pi}{N}$

where K depends on type of window used.

K=1 for rectangular, k=2 triangular…..

Therefore $\quad N \geq K \dfrac{2\pi}{w_2 - w_1}$

If analog freq are given, it must be converted in to Digital using $w = \Omega T$

Ex:

Apply the Hamming Window to improve the low pass filter magnitude response ontained in ex1:

$W_H(n) = 0.54 + 0.46\ Cos\left(\dfrac{2\pi n}{N-1}\right) \qquad$ for $|n| \leq \dfrac{N-1}{2}$

$= 0 \qquad$ else where

N = 2M+1 = 21

$W_H(0) = 1$	$W_H(6) = 0.39785$
$W_H(1) = 0.97749$	$W_H(7) = 0.26962$
$W_H(2) = 0.91215$	$W_H(8) = 0.16785$
$W_H(3) = 0.81038$	$W_H(9) = 0.10251$
$W_H(4) = 0.68215$	$W_H(10) = 0.08$
$W_H(5) = 0.54$	

Next these window sequence values are multipled with coefficients h(n), obtained in ex1, to obtain modified F.S Co eff h'(n).

h'(0) =0.25

h'(1) =0.22

h'(2) =0.14517

h'(3) =0.0608

h'(4) =0

h'(5) =0.02431

h'(6) =0.02111

h'(7) =-0.0086725

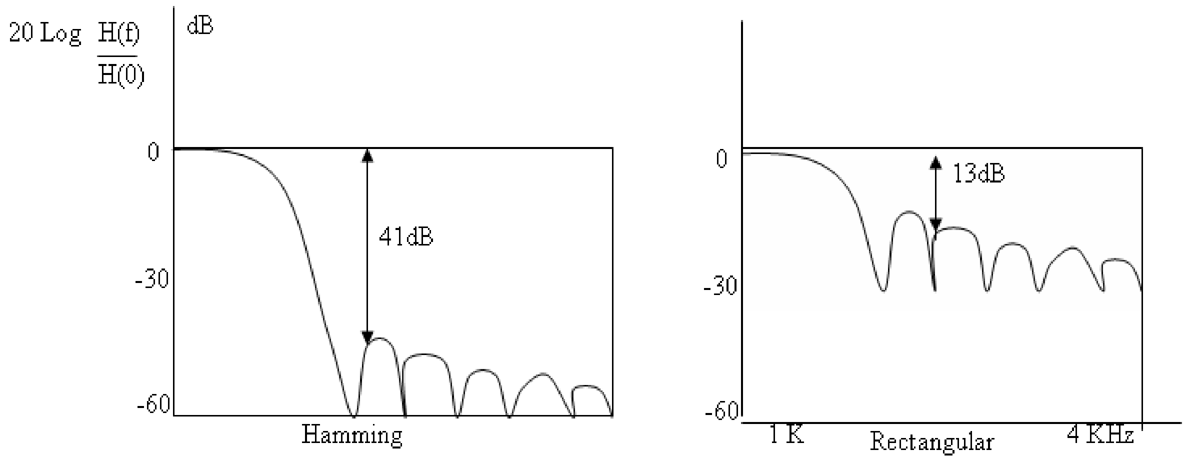
h'(8) =0

h'(9) =0.00256

h'(10) =0.00255

$$H'(z) = \sum_{i=0}^{2M} b'_i Z^{-i}$$

$b_i' = h'(i-M) \quad 0 \le i \le 20 \qquad h'(-n) = h'(n)$



Ex:

Find a suitable window and calculate the required order the filter to design a LP digital filter to be used A/D-H(Z)-D/A structure that will have a -3dB cutoff of at 30π rad/sec and an attenuation of 50dB at 45π rad/sec. the system will use a sampling rate of 100 samples /sec

Sol:

The desired equivalent digital specifications are obtained as

Digital ….. $w_1 = w_c = \Omega c T = 30\pi \frac{1}{100} = 0.3\,\pi \quad k_1 \ge -3dB$

$w_2 = \Omega 2T = 45\pi \frac{1}{100} = 0.45\,\pi \qquad k_2 \le -50dB$

1. to obtain a stop band attenuation of -50dB or more a Hamming window is shosen since it has the smallest transition band.

2. the approximate no. of points needed to satisfy the transition band requirement (or the order of the filter) can be found for w1 =0.3πrad &w2 = 0.45πrad, using Hamming window (k=2), to be

$$N \ge k \frac{2\pi}{w_2 - w_1} = \frac{2.2\pi}{0.45\pi - 0.3\pi} = 26.65$$

N = 27 is selected

Kaiser window

➢ The attractive property of the Kaiser window is that the side lobe level and main lobe width can be varied continuously by simple varying the parameter β. Also as in other window, the main lobe width can be adjusted by varying N.

➢ we can find out the order of Kaiser window, N and the Kaiser parameters β to design FIR filter with a pass band ripple equal to or less that Ap, a minimum stop band attenuation equal to or greater than As, and a transition width Wt, using the following steps:

Step 1 : Choose δ such that $\delta = Min(\delta_p, \delta_s)$

$\delta_s = 10^{-0.05As}$, [Prove $A_s = 20\log_{10}\frac{1+\delta p}{\delta s}$ => $A_s = -20\log\delta s$]

$$\delta_p = \frac{10^{0.05Ap} - 1}{10^{0.05Ap} + 1}$$

[Prove $A_p = 20\log_{10}\frac{1+\delta p}{1-\delta p}$

$$10^{0.05Ap} = \frac{1+\delta}{1-\delta}$$

$$(1-\delta)10^{0.05Ap} = 1+\delta$$

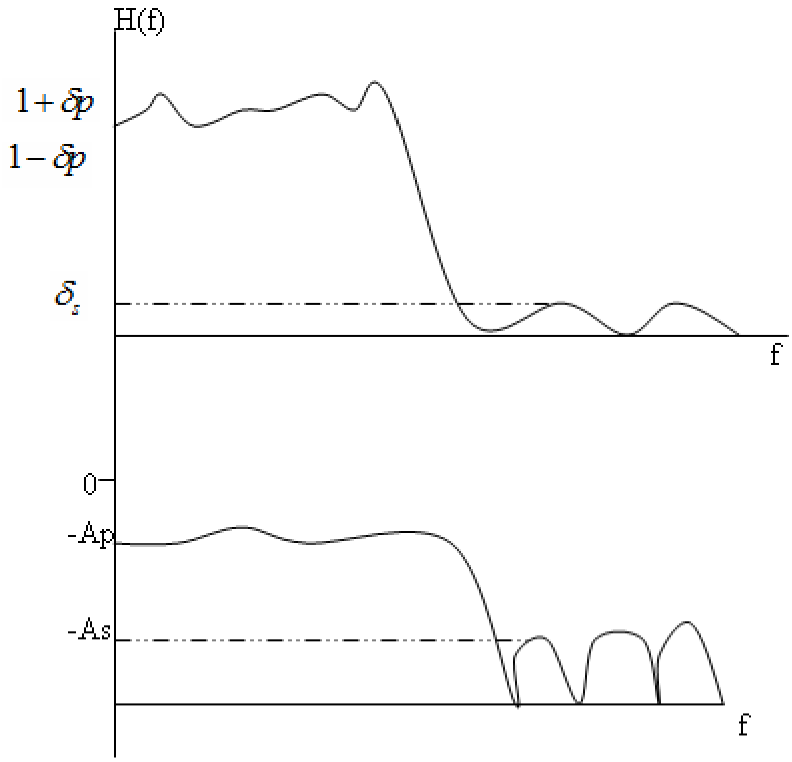
Therefore: solving above eq for δ, we get

$$\delta = \frac{10^{0.05Ap} - 1}{10^{0.05Ap} + 1}$$]

Step 2:

Calculate A_s using the shosen values

$A_{so} = -20\log\delta$



Step 3:

Calculate the parameter β as follows for

β = 0 for $A_{so} \leq 21$ dB

= $0.5842(A_{so} - 21)^{0.4} + 0.07886(A_{so} - 21)$ for $21 < A_{so} \leq 50$ dB

= $0.1102(A_{so} - 8.7)$ for $A_{so} > 50$ dB

Step 4:

Calculate D as follows

D = 0.9222 for $A_{so} \leq 21$ dB

$= \dfrac{As - 7.95}{14.36}$ for $A_{so} > 21$ dB

Step 5:

Select the lowest odd value of N satisfying the inequality

$$N \geq \frac{\Omega samD}{\Omega t} + 1$$

Wsam : Angular Sampling frequency

Ω sam : Analog Freq

$\Omega_t = \Omega_s - \Omega_p$ for LPF

$= \Omega_p - \Omega_s$ for HPF

$= \text{Min}[(\Omega_{p1} - \Omega_{s1}), (\Omega_{s2} - \Omega_{p2})]$ for BPF

$= \text{Min}[(\Omega_{s1} - \Omega_{p1}), (\Omega_{p2} - \Omega_{s2})]$ for BSF

-3dB cutoff freq Ω_c can ve considered as follows

$\Omega_c = \frac{1}{2}(\Omega p + \Omega s)$ for LPF & HPF

$\Omega_{c1} = \Omega_{p1} - \frac{\Omega t}{2}; \Omega_{c2} = \Omega_{p2} + \frac{\Omega t}{2}$ for BPF

$\Omega_{c1} = \Omega_{p1} + \frac{\Omega t}{2}; \Omega_{c2} = \Omega_{p2} - \frac{\Omega t}{2}$ for BSF

Ex:

Calculate the Kaiser parameter and the no. of points in Kaiser Window to satisfy the following lowpass specifications.

Pass band ripple in the freq range 0 to 1.5 rad/sec ≤ 0.1 dB

Minimum stop band attenuation in 2.5 to 5.0 rad /s $\geq$ 40 dB

Sampling frequency: 10 rad/s

Sol:

The impulse response samples can be calculated using h(n) = $\frac{1}{n\pi}[\sin \Omega_c nT]$

Where $\Omega c = \frac{1}{2}(1.5 + 2.5) = 2$rad/s

And the no. of points required in this sequence can be found as follows

Step1:

$$\delta s = 10^{-0.05(40)} = 0.01$$

$$\delta p = \frac{10^{0.05(0.1)} - 1}{10^{0.05(0.1)} + 1} = 5.7564 * 10^{-3}$$

Therefore we choose, $\delta = 5.7564 * 10^{-3}$

Step 2:

A_{so} = -20 log($5.7564 * 10^{-3}$) = 44.797 dB

Step 3 & 4:

β = 0.5842 (44.797 -21)$^{0.4}$ + 0.07886 (44.797 -21) = 3.9524

D = 2.566

Step 5:

$N \geq \frac{10(2.566)}{1} + 1 = 26.66$ => N=27

$$W_k(n) = \frac{Io\left[\beta\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right]}{Io(\beta)}$$

$$W_k(0) = \frac{Io(\beta)}{Io(\beta)} = 1$$

$$W_k(1) = \frac{Io\left[3.9524\sqrt{1-\left(\frac{2}{26}\right)^2}\right]}{Io(3.9524)} = \frac{Io(3.94)}{Io(3.9524)} = \frac{10.269}{10.3729} = 0.9899$$

9.OBJECTIVE PAPER-1

1)What is the parsval's theorem expression in DTFT :
$\sum_{n=-\infty}|x(n)|^2=(1/2\pi)\int_{-\pi}^{\pi}|X(w)|^2dw$

Match the following:

2) E = ∞, P = 0 a) power
3) E ≠ ∞, P = 0 b) Neither energy nor power
4) E = ∞, P ≠ 0, P ≠ ∞ c) Energy

Match the following

5) $e^{-t}u(t)$ a) power
6) u(t) b) Neither energy nor power
7) 1/√t c) Energy

8) x (n) = $6e^{j2\Pi n/4}$, what is the power of the signal
a) 36W b) 72W c) 18W d) none

Match the following: For a real valued sequence, the DTFT follow the properties as

9) Re [H (jw)] a) Real valued function of w
10) I_m[H(jw)] b) even function of w
11) F.T [even symmetric sequence] c) Imaginary valued function of w
12) F.T [odd symmetric sequence] d) odd function of w

13) x(n) = {4, 1, 3} h(n) = {2, 5, 0, 4} what is the output of the system.

a) {8, 22, 11, 31, 4, 12} b) {8, 22, 11, 31, 4, 12} c) {8, 22, 11, 31, 4, 12} d) none

14) y(n) = x(n) * h(n) then y^1 (n) = {0, 0, x(n), 0 } * { 0, h(n), 0 } is equal to
a) {0, 0, y(n), 0} b) {0, 0, 0, y(n), 0, 0} c) [0, 0, y(n), 0 } d) {0, y(n), 0, 0}

15)If x(n) and h(n) are having N values each, to obtain linear convolution using circular convolution, the number of zeros to be appended to each sequence is
a) N – 1 b) 2N – 1 c) N d) N + 1

16)W_4^9 = ?
a) – j b) + j c) + 1 d) -1

17) DFT [x^* (-n)] = ?
a) $X^*(K)$ b) $X^*(-K)$ c) $X^*(N-K)$ d) none

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	b	c	a	c	a	b	a	b	d	a	c	c	b	a	a	a

10.OBJECTIVE PAPER-2

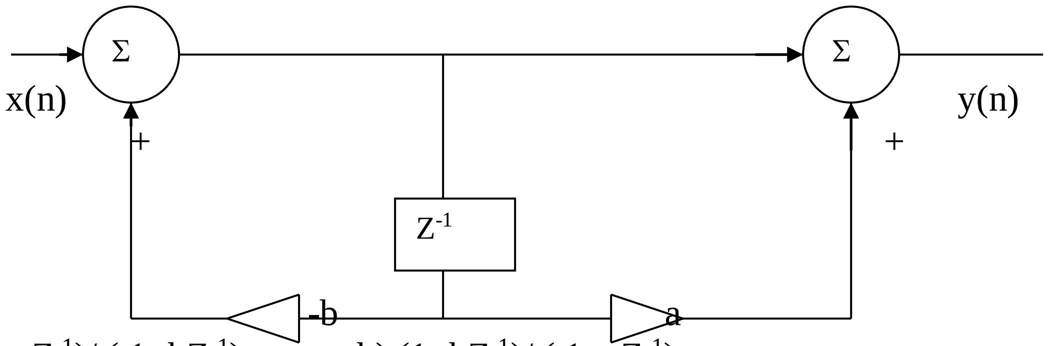
1) The region of convergence of the Z-transform of a unit step function is
a)|Z| >1 b) |Z|<1 c) (real part of Z) >0 d) (real part of Z) <0

2) The Z T of the function f(nT) = a^{nT} is
a) $Z/(Z-a^{T})$ b) $Z/(Z+a^{T})$ c) $Z/(Z-a^{-T})$ d) $Z/(Z+a^{-T})$

3) The Z T of the function $\sum_{k=0}^{\infty}\delta(n-k)$ is
a) (Z-1)/Z b) $Z/(Z-1)^2$ c) Z/(Z-1) d) $(Z-1)^2/Z$

4) The Z T of a signal is given by $X(Z)= Z^{-1}(1-Z^{-4})/(4(1-Z^{-1})^2)$ its final value is
a) ¼ b) 0 c) 1 d) infinity

5) Consider the system shown in fig. The transfer function Y(Z) / X(Z) of the system is



a) $(1+aZ^{-1})/ (1+bZ^{-1})$ b) $(1+bZ^{-1})/ (1+aZ^{-1})$
c) $(1+aZ^{-1})/ (1-bZ^{-1})$ d) $(1-bZ^{-1})/ (1+aZ^{-1})$

6) A linear discrete time system has the characteristic equation Z^3-0.8 Z=0, the system
a) is stable b) is marginally stable
c) is un stable d) stability cannot be assessed from the given information

7) The advantage of Canonic form realization is
a) smaller no of delay elements b) larger no of delay elements
c) hard ware flexibility d) none

8) y(n) = $\sum_{k=-2}^{3} akx(n-k)$ - $\sum_{k=1}^{5} bky(n-k)$ the minimum no of delay elements needed to realize the system is
a) 5 b) 10 c) 8 d) 11

9) Expand CSOS Ans: Cascaded form of second order section.
PSOS Ans: Parallel form of Second order section

10) To ensure a causal system, the total no of zeros must be less than or equal to the total number of poles (T / F)

1	2	3	4	5	6	7	8	9	10
a	a	c	c	a	a	a	c		T

11) The poles or zeros at the origin do effect the magnitude response (T / F)

12) All poles and zeros of a minimum phase system lie inside the unit circle (T / F)

13) To realize FIR filter
a) no feedback paths and forward path
b) no feedback paths and no forward path
c) feedback paths and no forward path
d) feedback paths and forward path

14) Find total no of complex multiplications using FFT for N=8: ___________

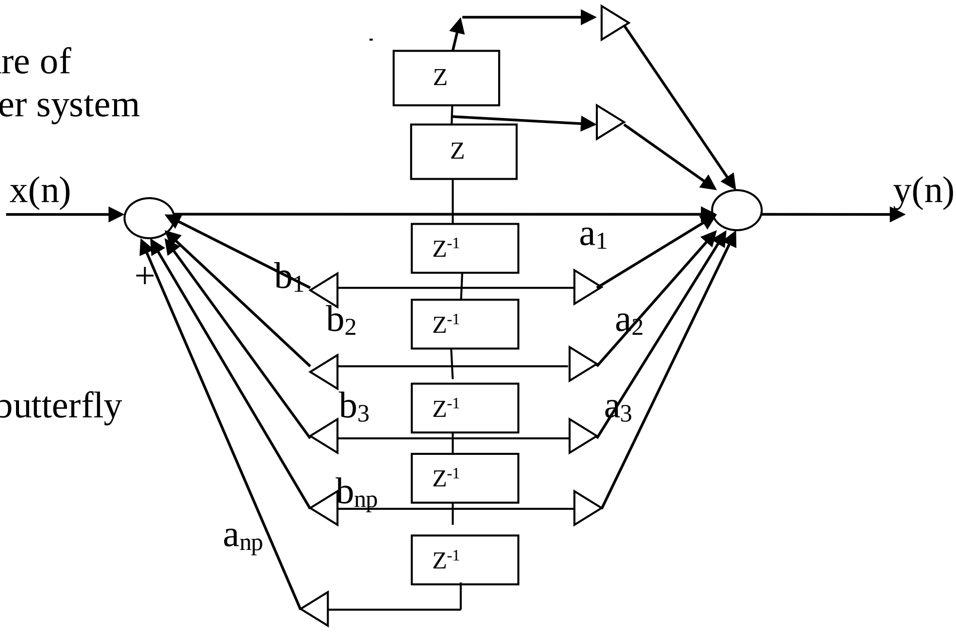
15) Find total no of complex additions using FFT for N=8: ___________

16) Find total no of real additions using direct DFT for N=8: ___________

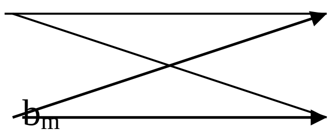
11	12	13	14	15	16
F	T	a	12	24	240

17) What is Z T of 2 $(3^{n)}$ u (-n-1): ____(-2)/(1-3z^{-1}_)___ or (-2z)/(z-3)___________

18) (2M) Show the structure of Direct form –II for 2^{nd} order system



19) Show the structure of butterfly



11.OBJECTIVE PAPER-3

State TRUE or FALSE

1) u(n) = $\sum_{K=0}^{\infty}\delta(n-k)$

2) x(n) = cos 0.5n is periodic sequence

3) Discrete-time sinusoidal signals with frequency that are separated by an integral multiple of 2π are identical

4) y(n) =x(-n) is time invariant

Match the following

5) $\sum_{-\infty}^{\infty}\lvert h(k)\rvert < \infty$	1 Zero input response
6) Impulse response of difference equation is	2 linear
7) y(n) = |x(n)|	3 Stable
8) y(n) = x(n^2)	4 Time invariant

CHOOSE THE CORRECT ANSWER

9) x(n) = Cos 0.125Πn , what is the period of the sequence

a) 8 b) 16 c) 125 / 2 d) none

10) y (n) = x (2n)

a) Causal b) Non-Causal c) Time invariant d) none

11) x(-n + 2) is obtained using following operartion

a) x (-n) is delayed by two samples b) x (-n) is advanced by two samples
c) x (n) is shifted left by two samples d) none

12) In situations where both interpolation and decimation are to be performed in succession, it is therefore best to

a) Interpolate first, then decimate b) Decimate first and interpolate
c) Any order we can perform d) none

1	2	3	4	5	6	7	8	9	10	11	12
T	F	T	F	3	1	4	2	B	B	A	a

13) The output of anti causal LTI system is

a) y (n) = $\sum_{K=0}^{\infty}h(k)x(n-k)$ b) y (n) = $\sum_{K=0}^{n}h(k)x(n-k)$

c) y (n) = $\sum_{-\infty}^{-1}h(k)x(n-k)$ d) y (n) = $\sum_{-\infty}^{\infty}h(k)x(n-k)$

14) $\delta(n-k) * x(n-k)$ is equal to

a) $x(n-2k)$ b) $x(n-k)$ c) $x(k)$ d) none

15) Given x(n) the $y(n) = x(2n – 6)$ is

a) x(n) is Compressed by 2 and shifted by 6 b) x(n) is Compressed by 2 and shifted by 3

c) x(n) is Expanded by 2 and shifted by 3 d) none

16) Decimation by a factor N is equivalent to

a) Sampling x(t) at intervals t_s / N b) Sampling x(t) at intervals t_sN

c) N fold increase in sampling rate d) none

17) In fractional delay, x(n-M/N), specify the order of operation.

a) Decimation by N, shift by M, Interpolation by N

b) Shift by M, Decimation by N and Interpolation by N

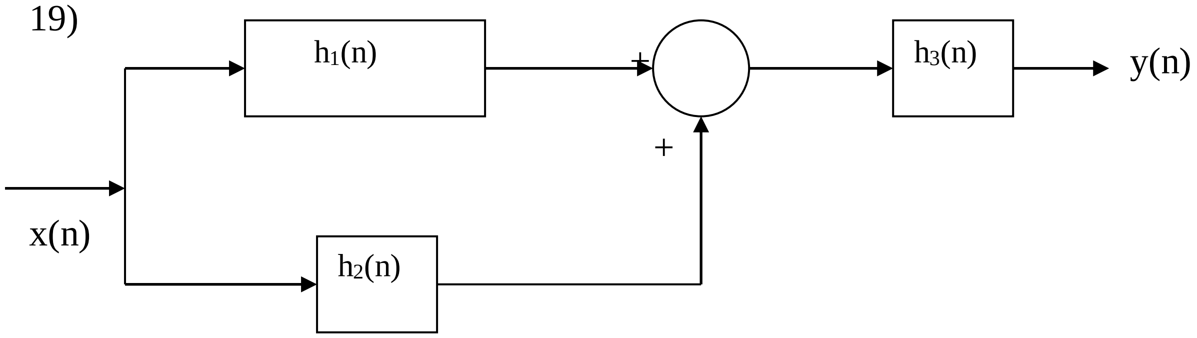
c) Interpolation by N, Shift by M and Decimation by N

d) All are correct

18) Given $g(n) = \{1, \underset{\uparrow}{2}, 3\}$, find $x(n) = g(n/2)$, using linear interpolation

a) 1, 0, 2, 0, 3 b) 1, 1, 2, 2, 3, 3 c) 1, 3/2, 2, 5/2, 3 d) none

19)



In the figure shown, how do you replace whole system with single block

a) $[h_1(n) + h_2(n)] * h_3(n)$ b) $h_1(n)h_3(n) * h_2(n)h_3(n)$

c) $[h_1(n) + h_2(n)] h_3(n)$ d) none

20 The h(n) is periodic with period N, x(n) is non periodic with M samples, the output y(n) is

a) Periodic with period N b) Periodic with period N+M

c) Periodic with period M d) none

13	14	15	16	17	18	19	20
C	A	B	B	C	C	A	A

12.**OBJECTIVE PAPER-4**

1) If x(n) = {-1, 0, 1, 2, 1, 0, 1, 2, 1, 0, -1} What is X(0)

a) 6 b) 10 c) 0 d) none

2) If x(n) = 1, $|n|\leq 2$
0, other wise
Find DTFT
a) sin(5w)/sinw b) sin(4w)/sinw c) sin(2.5w)/sin(0.5w) d) none of the above

3) If x(n)=h(n)=u(n), then h(n) is equal to
a) (n+1)u(n) b) r(n) c) r(n-1) d) none

4) if x ~ (n) = { 1,0,1,1} and h ~(n) = { 1, 2, 3,1} find y ~(n)
a) {6 , 6, 5, 4} b) {1, 2, 4, 4} c) {5, 4, 1, 0} d) None

5) x(n) = {4, 1, 3} h(n) = {2, 5, 0, 4} what is the output of the system.

a) {8, 22, 11, 31, 4, 12} b) {8, 22, 11, 31, 4, 12} c) {8, 22, 11, 31, 4, 12} d) none

6) y(n) = x(n) * h(n) then y^1 (n) = {0, 0, x(n), 0 } * { 0, h(n), 0 } is equal to
a) {0, 0, y(n), 0} b) {0, 0, 0, y(n), 0, 0} c) [0, 0, y(n), 0 } d) {0, y(n), 0, 0}

7) If x(n) and h(n) are having N values each, to obtain linear convolution using circular convolution, the number of zeros to be appended to each sequence is
a) N – 1 b) 2N – 1 c) N d) N + 1

8)W_4^9 = ?
a) – j b) + j c) + 1 d) -1

9) DFT [x^* (-n)] = ?
a) X^*(K) b) X^* (-K) c) X^*(N-K) d) none

10) If x(n)⇔X(K), then IDFT [X (K), X(K)] = ?
a) x (n / 2) b) 2x (n/2) c) ½ x (2n) d) none.

11) Both discrete and periodic in one domain are also periodic and discrete in other domain (T / F)

12) If h(n)= -h(-n) then H(K) is purely real (T / F)

13) Reversing the N point sequence in time is equivalent to reversing the DFT values (T / F)

14) FT of non periodic discrete time sequence is non periodic (T / F)

Match the following: For a real valued sequence, the DTFT follow the properties as

15) Re [H (jw)] a) Real valued function of w

16) I_m[H(jw)] b) even function of w

17) F.T [even symmetric sequence] c) Imaginary valued function of w

18) F.T [odd symmetric sequence] d) odd function of w

19) Write DFF & IDFT formulas. $X(k)=\sum_{n=0}^{n=N-1} \mathbf{x(n)W_n^{nk}}$

$x(n)=(1/N)\sum_{K=0}^{N-1} X(k)Wn^{nk}$

20) Total no of real multiplications in DFT is:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	C	A	A	C	B	A	A	A	A	T	F	T	F	B	D	A	C		$4n^2$

13.OBJECTIVE PAPER-5

Choose the Correct Answers

1. The Fourier transform of a finite energy discrete time signal, x(n) is defined as [ ]

a) $X(\omega)= \sum_{n=-\infty}^{\infty} x(n)\, e^{j\omega n}$ b) $X(\omega)= \sum_{n=-\infty}^{\infty} x(n)\, e^{\omega n}$

c) $X(\omega)= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$ d) $X(\omega) = \sum_{n=0}^{\infty} x(n)\, e^{-j\omega n}$

2. Inverse DFT (IDFT) of X(K) is x(n), where k=0,1,-----n-1. It is given as [ ]

a) $x(n) = \frac{1}{N}\sum_{n=0}^{N-1} X(k)\, e^{\frac{j2\pi kn}{N}}$ b) a) $x(n) = \frac{1}{N}\sum_{n=0}^{N+1} X(k)\, e^{\frac{j2\pi kn}{N}}$

c) $x(n) = \frac{1}{N}\sum_{n=0}^{\infty} X(k)\, e^{\frac{j2\pi kn}{k}}$ d) a) $x(n) = \frac{1}{N}\sum_{n=0}^{N} X(k)\, e^{\frac{j2\pi kn}{N}}$

3. A N – periodic sequence x(n) and its DFT x(k) are known. Then the DFT of x(n) = δ(n) will be

a) $e^{-j2\pi nk}$ b) 1 c) $e^{-j2\pi nok/N}$ d) $e^{-j2\pi nk}/N$ [ ]

4. If the length of sequence x(n) is L and h(n) is M then the length of o/p sequence of the circular convolution is [ ]

a) L+M b) L+M-1 c) L if L>M d) 2L if L=M

STATE TRUE OR FALSE

5. The DFT of a sequence is a continuous function of ω [ ]

6. The DFT of even sequence is purely imaginary and DFT of odd sequence is purely real [ ]

7. The circular shift of an N point sequence is equivalent to linear shift of its periodic extension [ ]

8. The multiplication of DFT of two sequences is equal to DFT of the linear convolution of two sequences [ ]

Fill in the blanks

9. The 4-point DFT of a sequence x(n) is ________

10. DFT of a sequence $x(n) = \delta(n-n_0)$ is __________

11. An N point sequence is called ________________ if it is antisymmetric about point zero on the circle

12. The two methods of sectioned convolution are ________________ & ________________

13. DFT of multiplication of two sequences DFT $\{x_1(n) \quad x_2(n)\}$ = ______________________

14. DFT of even sequence is X(k)= _________________________& DFT of odd sequence is X(k) = ________________________

15. To get the result of linear convolution with circular convolution of sequence x(n) & h(n), the sequences should extended to the length of ___________________

16. Match the following

1 DFT [$x_1(n)\, x_2(n)$]	a) X (N-K)
2. DFT [$x^*(n)$]	b) $\frac{1}{N}$ [$X_1(k) \otimes X_2(k)$]
3. DFT [$x((-n))_N$]	c) $X^*(N-K)$
4. $X_1(k)\, X_2^*(k)$	d) $x_1(n) \otimes x_2(n)$
	e) $x_1(n) \otimes x_2^*(-n)$

1=	2=	3=	4=

17. Show that the given sequence x(n) = { 1,-2,3,2,1,0} for the following conditions using concentric circles.

a) x(-n) b) x(2-n) (2M)

18. Compute 4-point DFT of a sequence x(n) = {1,2,0,2} (2M)

14.OBJECTIVE PAPER-6

MULTIPLE CHOICES

1. In Impulse invariant transformation, the mapping of analog frequency Ω to the digital frequency is

a) one to one b) many to one c) one to many d) none

2. The digital frequency in bilinear transformation is

a) $w = 2\tan^{-1}(\Omega T_s/2)$ b) $w = \tan^{-1}(\Omega T_s/2)$

c) $w = 2\tan^{-1}(\Omega T_s)$ d) $w = 2\tan^{-1}(\Omega/2)$

3. Which technique is useful for designing analog LPF

a) Butter worth filter b) Chebyshev filter

c) Both a and b d) none

4. Which filter is more stable?

a) Butter worth b) Chebyshev c) none

5. As Ω increases , the magnitude response of LPF approaches with

a) –20Ndb/oct b) –6Ndb/oct c) –10Ndb/dec d) none

6. Using Impulse invariant technique the pole at $S= S_P$ is mapped to Z-plane as

a) $Z=e^{-S_P T_s}$ b) $Z=e^{(S_P T_s)}$ c) $Z=e^{S_P}(T_s)$ d) None

TRUE or FALSE

7. The disadvantage of Chebyeshev filter is less transition region
8. The advantage of Butter worth filter is flat magnitude response.
9. for the given same specifications order of the Chebyshev filter is more than Butterworth filter
10. Poles of Butterworth filter lies on circle.

1	2	3	4	5	6	7	8	9	10
B	A	C	A	B	B	F	T	F	T

FILL IN THE BLANKS

11.The Butterworth LPF of order N is defined as: $1/(1+(s/j\Omega_c)^{2N})$

12.For N=3 what are the stable Butter worth angles :$120^0,180^0,240^0$
13. –0.5db convert in to gain equivalent =0.994

14.Let $S_{1,2} = 2076\Pi e^{\pm j144°}$ $H_a(S)=$ $k/(s^2-10552.7s+(2076\pi)^2$ (2M)

15.Given $\Omega_s = 2000$; $T_s = 10^{-4}$; $\Omega_s^* = 2006$

16.Using Bi-linear transformation, the pole at $S = S_p$ is mapped into Z-plane using (2M)

$Z=1-(2+S_pT_S)/(2-S_pT_s)$

17.Given allowable ripples in Pass band is –3 dB, the value of μ is 0.997 (2M)

15.OBJECTIVE PAPER-7

Choose the correct Answer

1. In impulse invariant transformation the mapping of analog frequency Ω to digital frequency ω is [ ]
a) one to one b) many to one c) one to many none

2. The digital frequency in Bi –linear transformation is [ ]
a) $\omega = 2\tan^{-1}(\Omega T/2)$ b) $\omega = \tan^{-1}(\Omega T/2)$
c) $\omega = 2\tan^{-1}(\Omega T)$ d) $\omega = 2\tan^{-1}(\Omega /2)$
3. Using bilenear transformation for T = 1sec the pole p_k is in S- Plane is mapped to Z – plane using [ ]
a) $S = 2\left[\frac{1+z^{-1}}{1-z^{-1}}\right]$ b) $S = \frac{1+z^{-1}}{1-z^{-1}}$ c) $S = \left[2\frac{1+z^{-1}}{1-z^{-1}}\right]$ d) $S=\frac{Z+1}{Z-1}$

4. The normalized magnitude response of chebyshev type – I filter has a value of _____________ at cut off frequency are [ ]
a) $\frac{1}{\sqrt{1+\varepsilon^2}}$ b) $\frac{1}{\sqrt{1+\varepsilon}}$ c) $\frac{1}{\sqrt{1-\varepsilon^2}}$ d) $\sqrt{1+\varepsilon^2}$

5. For high pass analog filter the transformation used is [ ]
a) $S \to S/\Omega$ b) $S \to \Omega/S$ c) $S \to S/\Omega_c$ d) $S \to \Omega_c/S$

6. The magnitude response of Type I – chebyshev LPF is given by [ ]
a) $|H_a(\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N(\Omega/\Omega_c)}$ b) $|H_a(\Omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2(\Omega/\Omega_c)}$
c) $|H_a(\Omega)|^2 = \frac{1}{1-\varepsilon^2 C_N^2(\Omega/\Omega_c)}$ d) $|H_a(\Omega)| = \frac{1}{1+\varepsilon^2 C_N(\Omega/\Omega_c)}$

7. The width of main lobe in rectangular window spectrum is [ ]

a) $2\pi/N$ b) $4\pi/N$ c) $8\pi/N$ d) $16\pi/N$

8. The width of main lobe in Hamming window is [ ]

a) $4\pi/N$ b) $2\pi/N$ c) $8\pi/N$ d) $16\pi/N$

9. The frequency response of rectangular window $W_R(w)$ is [ ]

a) $\frac{Sinwn/2}{Sinw/2}$ b) $\frac{Sinwn/2}{Sinw}$ c) $\frac{Sinwn/2}{Sinwn}$ d) $\frac{Sinwn/2}{Sinwn/2}$

10. In …………………. Window spectrum the width of main lobe is double that of rectangular window for same value of N [ ]

a) Hamming window b) Kaiser window c) Blackman window d) none

State TRUE or FALSE

11. The disadvantage of chebyshev filter is less transition region [ ]

12. For chebyshev Type 2 filter ripples are present in pass band and stop band [ ]

13. The advantage of Butter worth filter is flat magnitude response. [ ]

14. for cheby shev Type 1 filter equi–ripples are present only in pass band. [ ]

15. For same specifications, the order N of chebyshev filter is less compared to Butter worth filter. [ ]

16. FIR filter have non-linear phase characteristics. [ ]

17. FIR filters are non – recursive and stable filters. [ ]

18. The design of Digital transformation H (z) of IIR filter is direct and FIR is indirect [ ]

19. Poles of chebyshev filter lies on circle [ ]

20. In FIR filter with constant phase delay the impulse response is symmetric [ ]

16.OBJECTIVE PAPER-8

CHOOSE THE CORRECT ANSWER

1. The DTFT of a sequence x(n) is [ ]

a) $\sum_{n=-\alpha}^{\alpha} x(n)e^{-jwn}$ b) $\sum_{n=-\alpha}^{\alpha} x(n)e^{jwn}$ c) $\int_{-\alpha}^{\alpha} x(n)e^{jwn}dw$ d) $\int_{-\alpha}^{\alpha} x(n)e^{-jwn}dw$

2. DTFT of e^{jwon} x(n) is [ ]

a) x[e $^{j(w-w_o)}$] b) x[e $^{j(w+w_o)}$] c) x[e $^{j(ww_o)}$] d) x[e $^{j(-w-w_o)}$]

3. DTFT of $x_1[n]$ * $x_2[n]$ is [ ]

a) $X_1[w]\ X_2[w]$ b) $\frac{1}{N}X_1[w]\ X_2[w]$ c) $X_1[w] * X_2[w]$ d) $\frac{1}{N}X_1[w] * X_2[w]$

4. The smallest value of N for which x(n +N) = x(n) holds is called [ ]

a) Fundamental period b) Fundamental frequency c) fundamental signal d) None

5. DFS of real part of periodic signal is [ ]

a) $X_e(K)$ b) X_o (K) c) $X_R(K)$ d) $X_{Im}(K)$

6. Expression for DFT is [ ]

a) $\sum_{n=0}^{N-1} x(n)W_N^{Kn}$ b) $\sum_{n=0}^{N-1} x(n)W_N^{-Kn}$ C) $\sum_{K=0}^{N-1} x(n)W_N^{Kn}$ d) $\sum_{n=0}^{N-1} x(n)W_N^{-Kn}$

7. DFT of $x_1[n]\ x_2[n]$ is [ ]

a) $\frac{1}{N}$ $X_1[K] * X_2[K]$ b) $\frac{1}{N}$ $X_1[K] + X_2[K]$ c) $X_1[K] * X_2[K]$ d) $X_1[K] + X_2[K]$

8. If M & N are the lengths of x(n) & h(n) then length of x(n) * h(n) is [ ]

a) M+ N –1 b) M + N +1 c) max (M,N) d) min (M,N)

9. Zero padding means [ ]

a) increasing length by adding zeros at the end of sequence
b) Decreasing length by removing zeros at the end
c) Inserting zeros in between the samples d) None of the above

II STATE TRUE OR FALSE

10. The F.T of discrete signal is a discrete function of ω [ ]

11. In a discrete signal x(n), if x(n) =x(-n) then it is called symmetric signal [ ]

12. The F.T of the product of two time domain sequence is equivalent to product of their F.T [ ]

13. The DFT of a signal can be obtained by sampling one period of FT of the signal [ ]

14. DFS is same as DTFS [ ]

17.OBJECTIVE PAPER-9

CHOOSE THE CORRECT ANSWER

1. Power signal is

a) Periodic b) aperiodic c) Continuous d) none [ ]

2. W_N^{nK} is

a) $e^{\frac{-j2\Pi K}{N}}$ b) $e^{-j2\Pi nK}$ c) $e^{\frac{-j2\Pi Kn}{N}}$ d) $e^{\frac{2\Pi Kn}{N}}$ [ ]

3. When the sequence is circularly shifted in time domain by 'm' samples i.e. $x((n-m))_N$ then on applying DFT, it is equivalent multiply sequence in frequency domain by

a) $e^{\frac{j2\Pi Km}{N}}$ b) $e^{\frac{-j2\Pi Km}{N}}$ c) $e^{-j2\Pi Km}$ d) $e^{\frac{-2\Pi Km}{N}}$ [ ]

4. Multiplication of sequence in time domain, on apply DFT, it corresponds to circular convolution in frequency domain and is given as

a) $x_1(n)\ x_2(n) \xleftrightarrow{DFT} X_1(K) \quad X_2(K)$

b) $x_1(n)\ x_2(n) \xleftrightarrow{DFT} X_1(K)X_2(K)$

c) $x_1(n) * x_2(n) \xleftrightarrow{DFT} X_1(K) \quad X_2(K)$

d) $x_1(n)\ x_2(n) \xleftrightarrow{DFT} \sum_{K=0}^{N-1} X_1(K)X_2(K)$

5. Linear convolution of two sequences N_1 and N_2 produces an output sequence of length

a) $N_1 - N_2 +1$ b) $N_1 + N_2 -1$ c) $N_1 + N_2 +1$ d) $2N_1 - N_2 +1$[ ]

FILL IN THE BLANKS

6. The basic signal flow graph for butterfly computation of DIT-FFT is

7. The Fourier transform of discrete time signal is called ………………………

8. FFT's are based on the ………………………….. of an N-point DFT into successively smaller DFT's.

9. The Fourier transform of x(n)*h(n) is equal to …………………………..

10. Appending zeros to a sequence in order to increase the size or length of the sequence is called ……………………..

11. In N-point DFT using radix 2 FFT, the decimation is performed …………… times.

12. In 8-point DFT by radix 2 FFT, there are …………… stages of computations with …………………….. butterflies per stage.

13. If DFT of x(n) is X(K), then DFT of $W_N^{\ln}$x(n) is …………………….

ANSWER THE FOLLOWING

14. What are the differences between linear and circular convolution?

15. How many multiplications and additions are required to compute N-point DFT using radix 2 FFT

16. How many multiplications and additions are required to compute N-point DFT

17. What is the expression for N-point DFT of a sequence x(n) ?

18. What is the expression for N-point IDFT of a sequence X(K) ?

19. Define Aliasing error.

20. What is meant by Inplace computation.

18.OBJECTIVE PAPER-10

1. How we can calculate IDFT using FFT algorithm. (2M)

2. Draw the basic butterfly diagram for DIF algorithm.

3. Z[x(n)] = X(Z) then Z{x(n-m)} = …………………………………..

4. Define convolution property in Z-Transform.

5. Find the Z-Transform and ROC for the signal $x(n) = a^n u(n)$.

6. Find the Z-Transform and ROC for the signal $x(n) = - a^n u(-n-1)$.

7. Write the initial value theorem expression.

8. Z{δ(n)} = ……………………..

9. Find inverse Z-Transform for X(z) = $\frac{Z}{Z-1}$ when ROC is Z<1

10.What are the differences and similarities between DIT and DIF algorithms. (2M)

11.Give the Direct form II realization for second order system.

12.Give the Direct for I realization for second order system.

13.What is the relationship between Z-Transform and Fourier transform.

STATE TRUE OR FALSE:

14.ROC of a causal signal is the exterior of a circle of some radius r. [ ]

15.ROC of a anti causal signal is the exterior of a circle of some radius r. [ ]

16.ROC of a two sided finite duration frequency is entire Z-plane. [ ]

17.Direct form I required less no.of memory elements as compared to Canonic form.[ ]

18.A linear time invariant system with a system function H(Z) is BIBO stable if and only if the ROC for H(Z) contains unit circle. [ ]

19.OBJECTIVE PAPER-11

ANSWER THE FOLLOWING

1. What are the advantages of digital filter over analog filter.

2. What is the relation between analog and digital radiant frequency in Impulse Invariance design..

3. What is the relation between analog and digital radiant frequency in Bilinear transformation design.

4. What are the drawbacks with Impulse Invariance method?

5. What is the disadvantage with Bilinear transformation technique.

6. What is the relation between S & Z in Bilinear transformation?

7. Mention any two techniques to design IIR Filter from analog filter.

8. What are the differences between Chebyshev type I and type II.

9. What are the differences between Butterworth & Chebyshev filter.

10. What is the expression for magnitude squared frequency response of Butterworth analog filter?

11. What is the expression for magnitude squared frequency response of Chebyshev analog filter?

TRUE OR FALSE

12. Poles of Butterworth filter lies on circle. [ ]

13. Poles of Chebyshev filter lies on circle. [ ]

14. Transition bandwidth for Chebyshev is more as compared to Butterworth filter. [ ]

15. Butterworth filters are all pole filters. [ ]

16. Chebyshev, type-II are all pole filters. [ ]

17. Chebyshev, type II filter exhibit equiripple behavior in the pass band and monotonic characteristic in the stopband. [ ]

18. Chebyshev, type I filter exhibit equiripple behavior in the pass band and monotonic characteristic in the stopband. [ ]

19. Butterworth filter exhibit monotonic behavior both in passband and stopband. [ ]

20. For the given specifications order of the Chebyshev filter is more as compared to Butterworth filter. [ ]

20. OBJECTIVE PAPER-12

1. Define the following

a. Time variant system with an example(Equation)

b. Power signal with an example

c. Dynamic system

d. Recursive System

e. Non Recursive system

2. Give the example for FIR and IIR systems.

3. Give an example of Causal system

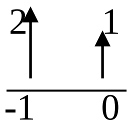
4. Write the condition to test the Linearity of the system

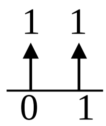
5. Plot $y(n) = x(n-2)$ Give $x(n) = \{1,2,\underset{\uparrow}{3},5,6\}$

6. Resolve the signal into impulse $x(n)=\{\underset{\uparrow}{4},5,4,4\}$ ------ 2 Marks

7. Give the expression for Convolution sum y(n)=

8. Find the Convolution Sum Graphically with all the steps-------3 Marks

x(n)= 

h(n)= 

9. Write the properties of Convolution Sum --------2 Marks

10. Write the expression for X(n) in terms of impulses

11. Write the necessary condition for the stability of the system

12. Write the general form of Difference equation

21.OBJECTIVE PAPER-13

State TRUE or FALSE

1. In direct –form II realization the number of memory locations required is more than that of direct form –I realization [ ]

2. An LTI system having system function H(z) is stable if and only if all poles of H(z) are out side the unit circle. [ ]

3. The inverse Z – transform of z/z-a is a^n u(n) [ ]

4. Digital filters are not realizable for ideal case. [ ]

5. As the order of Butter worth filter increases than the response is closer to ideal filter response. [ ]

Answer the following

6. Find the transfer function H(z) of the given difference equation
Y(n) = 0.7 y(n-1) – 0.12y(n-2) + x(n-1) + x(n-2)

7. Indicate the poles and zeros of the given system and also check the stability of the system

$$H(z) = \frac{z(z+1)}{(z-0.2)(z-0.4)(z+0.5)}$$ (2M)

8. Realize the given system function H(z) using direct form –II

$$H(z) = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}$$ (2M)

9. Realize the given system function H(z) using cascade form (2M)

H(z) = $\frac{1}{(1+0.5z^{-1})(1-0.5z^{-1})}$

10. Find the inverse z-transform of x(z) = $\frac{z}{(z-2)(z-3)}$ using partial fraction method. (2M)

11. Using cauchy residue method find the inverse z- transform of

x(z) = $\frac{z}{(z-1)(z-2)}$ for ROC :|z| >2 (2M)

12. Mention the two conditions to realize any digital filter

13. Draw the Magnitude response of Low Pass Butter Worth filter.

14. The order of the Butter Worth filter is obtained by using the formula N≥ ________________________

15. The cut- off frequency Ω_c is obtained by using the formula ______________________________

22.OBJECTIVE PAPER-14

Fill in the Blanks

1. The expansion of FFT is _______________

2. The main advantage of FFT is _____________

3. The number of multiplications needed in the calculation of DFT using FFT with 32-point sequence = ________________

4. __________________ number of additions are required to compute N – pt DFT using radix –2 FFT.

5. What is decimation in time algorithm.

State TRUE or FALSE

6. For DIT –FFT algorithm the input is bit reversed and the output is in natural order [ ]

7. By using radix –2 DIT –FFT algorithm it is possible to calculate 6-point DFT.[ ]

8. $W_N^{NK} = 1$ [ ]

9. $W_{N/2}^{NK} = 1$ [ ]

10. In DIT –FFT, the input sequence is divided into smaller subsequences [ ]

Answer the following

11. Calculate the DFT of the sequence x(n)={1,0,0,1} using DIT –FFT (2M)
12. Draw the Butterfly diagram for 8-point DFT using DIT –FFT algorithm (2M)

13. Find IDFT of the sequence X(k) = { 10, 0, 10, 0} (2M)

14. Write the steps for the calculation of IDFT using DIT –FFT (2M)

15.Write the values of the following

a) W_8^0 b) W_8^2 c) W_8^3 d) W_8^5

23.OBJECTIVE PAPER-15

CHOOSE THE CURRECT ANSWER

1. y(n)=x(2n) is a _____________ system [ ]

a) time invariant b) causal c) non causal d) none

2. y(n) = $nx^2(n)$ is a ____________system [ ]

a) Linear b) Non-linear c) time-invariant d) none

3. y(n)= x(n) +x(n-1) is a ____________ system [ ]

a) Dynamic b) Static c) time variant d) None

4. x(-n+2) is obtained by which of the following operations [ ]

a) x(-n) is shifted left by 2 samples b) x(-n) is shifted right by 2 samples

c) x(n) is shifted left by 2 samples d0 none

5. The necessary and sufficient condition for causality of an LTI system is [ ]

a) h(n) =0 for n=0 b) h(n) =0 for n>0 c) h(n) =0 for n<0

d) none

6. The convolution of two sequences x(n) =h(n) = {1, 2, -1} [ ]

a) { 1,4,2,-4,1} b) {1,-4,1,2,4} c) { 1,1,2,-4,4}

d) { 4,-4,2,1,1}

II STATE TRUE OR FALSE

7. An IIR system exhibits an impulse response for finite interval [T/F]

8. If the energy of a signal is infinite then it is called energy signal [T/F]

9. Static systems does not require memory [T/F]

10. A linear system is stable if its impulse response is absolutely summable[T/F]

III Answer the following:

11. The average power of a discrete time signal with period N is given by ___________

12. The convolution sum of causal system with causal sequence is ____________

13. Give the graphical representation of the following discrete signals.

i) $x(n) = (5-x) [4(x) – 4(x-3)\}$

ii) $x(n) = -0.5\delta(n+1) + 0.5\delta(n) – 0.75\delta (n-2)$

14. $x(n) = \{3, -2, 1,0,-1\}$ show for $x(-n)$ (1M)

15. If $x(n) = \{1,2,-2,-1\}$ show for $x(n-2)$ & $X(-n+2)$ (2M)

16. Find the convolution of $u(n) * u(n-2)$ (1M)

17. If the impulse response $h(n) = 2^n u(-n)$ then determine the corresponding system is causal or stable. (1M)

18. Test the given discrete system for linearity , causality and time invariance
$h(n) = n\, e^{|x(n)|}$ (2M)

ASSIGNMENT

1 (a) Draw the frequency response of N-point rectangular window.
(b) Design a fifth order band pass linear phase filter for the following specifications.
i. Lower cut-off frequency = 0.4 πrad/sec
ii. Upper cut-off frequency = 0.6 πrad/sec
iii. Window type = Hamming
Draw the filter structure. [4+12]

2) Design a band pass filter to pass frequencies in the range 1-2 radians/second using Hanning window N=5. Draw the filter structure and plot its spectrum. [16]

3) (a) Compare the performances of rectangular window, hamming window and Keiser window
(b) The desired response of a low pass filter is
Hd(ej!) = _ e−j3!, −3π _ ω _ 3π/4
0 , 3π/4 _ |ω| _ π
Determine H(ej!) for M=7 using a Hamming window. [6+10]

4) (a) Design a linear phase low pass filter with a cut-off frequency of π/2 radians/seconds. Take N=7
(b) Derive the magnitude and phase functions of Finite Impulse Response filter when
i. impulse response is symmetric & N is odd
ii. impulse response is symmetric & N is even. [8+8]

5) (a) Design a low pass filter by the Fourier series method for a seven stage with cut-off frequency at 300 Hz if ts = 1msec. Use hanning window.
(b) Explain in detail, the linear phase response and frequency response properties of Finite Impulse Response filters. [8+8]

6) (a) Outline the steps involved in the design of FIR filter using windows.
(b) Determine the frequency response of FIR filter defined by y(n) = 0.25x(n)+ x(n-1)+ 0.25x(n-2). Calculate the phase delay and group delay. [8+8]

7) (a) Define Infinite Impulse Response & Finite Impulse Response filters and com-pare.

(b) Design a low pass Finite Impulse Response filter with a rectangular window for a five stage filter given: Sampling time 1 msec; fc = 200Hz.Draw the filter structure with minimum number of multipliers. [6+10]

ASSIGNMENT

1) a) What are the advantages of Multirate signal processing?

b) Differentiate between Decimator and Interpolator?

2) Prove that spectrum of down sampler is sum of M uniformly shifted and stretched version of $X(e^{jw})$ scaled by a factor 1/M and also discuss the aliasing effect?

3) State and prove any one identity property in down sampler and any one identity property in up sampler?

4) Let x(n)={1,3,2,5,-1,-2,2,3,2,1},find

a) Up sample by 2 times and down sample by 4 times

b) Down sample by 4 times and up sample by 2 times c) Justify why these outputs are not equal.

24.Reference

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2 Discrete time signal processing by Alan V Oppenheim Ronald W schafer

3 Digital signal processing by MITHRA

4 Digital signal processing by Tharun kumar rawat

5 Analog and Digital Signal Processing by Ashok Ambardar